Abstract Book of:
The 2nde International Workshop on Advance in Nonlinear Dynamical Systems, Complex Networks and Applications (DySyX'2025) and Python Day

22-24, May 2025, FS Kenitra, Morocco

M. A. Aziz Alaoui, Arnaud Ducrot, Radouane Yafia

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1 Editor's introduction

1.1 About DySyX'2025

DySyX'2025 is a two days workshop (May 22-23, 2025) organized by Faulty of Sciences of Kénitra (LAGA), Ibn Tofail University and Le Havre Normandie University (LMAH). It will focus on recent advances in dynamical systems, complex networks, and applications in some fields of science and engineering. This meeting will provide a joint forum for leading applied mathematicians and scientists actively working in the diverse fields of life sciences in order to exchange ideas, advance the knowledge of research, discuss future research challenges, and explore opportunities for collaborative exchange and education. There will be several guest talks covering such recent trends. This workshop is organized in the frame of the PHC-Toubkal project 2025.

1.2 About Python Day

is a training day on Python software and its applications to complex network in epidemiology (eight hours) which be held on May 24, 2025. By Prof. Cyrille Bertelle from Le Havre Normandie University, France.
Details in French language: Implémentation de modèles épidémiques sur réseaux avec Python Cette formation se base sur les modèles épidémiologiques de type SIR et leurs variantes et vise à les implémenter et les simuler en Python. A partir de modèles classiques basiques, de type SIRS ou SEIRS, nous regardons comment il est possible de les étendre afin qu'ils répondent d'une manière plus précise aux problématiques qui avaient été évoqués notamment lors de la crise Covid-19, à savoir : prise en compte du confinement, de la vaccination et de la saturation des hôpitaux. Un autre aspect des crises épidémiques et notamment de la crise du Covid-19 est la propagation entre villes, régions ou pays. Une approche adaptée pour cette problématique consiste à déployer les modèles sur des réseaux où chaque noeud du réseau représente un espace à l'échelle considérée (ville, région, pays) où un modèle EDO est considéré comme suffisant. Les effets spatiaux sont ici représentés par des mobilités entre les noeuds du réseaux.
Une technique de déploiement d'un modèle épidémiologique sur réseaux sera implémentée avec Python et ses librairies Numpy, SciPy et Matplotlib permettant d'avoir des performances et des visualisations intéressantes.
Durée proposée (8h) :
- 2h de présentation synthétique dur Python, Numpy, SciPy et Matplotlib avec supports sous la forme de Jupyter Notebooks.
- 2h pour étudier et simuler en Python des extensions des modèles épidémiologiques traitant des problèmes de confinement, vaccination, saturation des hôpitaux.
- 2h pour présenter le déploiement des modèles épidémiologique sur réseaux et montrer une

implémentation en Python.
- 2h laissés aux participants pour manipuler le modèle en réseau en l'adaptant aux différentes extensions.
Dans ces volumes proposés, les participants peuvent démarrer des mises en oeuvre sur Python. Une analyse plus approfondie des simulations permettant de mieux maîtriser les paramètres de contrôle des processus nécessiterait un volume horaire plus important ou encore la réalisation d'un projet à faire chez eux.

2 Conference co-chairs

Prof. Radouane Yafia, Ibn Tofail University, Morocco
Prof. Aziz Alaoui, Le Havre Normandie University, France
Prof. Arnaud Ducrot, Le Havre Normandie University, France

3 Organizing committee

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5 Plenary Speakers

- **Arnaud Ducrot,** Le Havre Normandie University, France
  Title: Mathematical modelling and analysis of the adaptive dynamics in mosquito populations: Uniform persistence of malaria infection.

- **Ezzinbi Khalil,** Cadi Ayyad University, Morocco
  Title: Center manifold and stability in critical cases for some partial functional differential equations.

- **Griette Quentin,** Le Havre Normandie University, France
  Title: Asymptotic behavior of an epidemic model with infinitely many variants.

- **Redouane Qesmi,** Sidi Mohamed Ben Abdellah University, Morocco
  Title: State-Dependent Delays: A Gateway to Complex Dynamics.

- **Ekrem Savas,** Usak University, Turkey
  Title: Generalized ideal statistically convergent functions in intuitionistic fuzzy n-normed space

- **Ousmane Seydi,** Le Havre Normandie University, France
  Title: Age-structured plant invasion dynamics across spatially explicit landscapes.

- **Abdessamad Tridane,** United Arabic Emirates, UAE
  Title: A data-driven Mathematical Modelling Approach for Malaria in Africa.

- **M.A. Aziz Alaoui,** Le Havre Normandie University, France
  Title: Modélisation et Analyse des Systèmes Complexes.

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Mathematical modelling and analysis of the adaptive dynamics in mosquito populations: Uniform persistence of malaria infection

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Abstract: In this talk, we present a mathematical model for the spread of malaria, incorporating key factors such as human populations, mosquito behavior, and the mosquitoes' plasticity and adaptation to control measures like widespread insecticide-treated mosquito nets and indoor residual spraying. Through analysis of the model, we identify and describe the convergence and persistence properties of the solutions, using a small parameter that represents the interactions between mosquitoes in relation to their activity patterns. In our analysis, we extend some ideas from the theory of uniform persistence to the case of semiflow without dissipativity.

Keywords:
2020 Mathematics Subject Classification:
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References

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Mathematics Art: Poincaré's Tetratypology, or Singularities in Dynamical Systems

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Abstract: Singularities reveal the character of what is unique in its kind, what is original, strange or unusual. In mathematics, although there is no formal definition, the notion of "singular" is opposed to that of "generic". It's usually a point, a curve, a surface, a value or a case in which a certain mathematical object is not well defined. We discuss this in this talk, explaining why "The singularity is not necessarily dangerous in everything". On the contrary, Singularities are those little things that lead to big things, that form the skeleton of a system. To do this, we draw a parallel between Poincaré and Van Gogh, focusing on Poincaré's tetratypology (his classification of the singular points of a dynamical system), which he geometrized by setting up "mathematical objects", unknowingly illustrated by Van Gogh (in his famous painting: "Starry Night : La nuit etoilee", 1889), objects that immediately catch the eye: the singularities of the system.

Mathématiques et Art : Tétratypologie de Poincaré, ou Singularités en Systèmes Dynamiques

Résumé : Les singularités révèlent le caractère de ce qui est unique en son genre, de ce qui est original, étrange ou insolite. En Mathématique, bien qu'il n'y ait pas de définition formelle, la notion de singulier s'oppose à celle de générique . C'est en général un point, une courbe, une surface, une valeur ou un cas dans lequel un certain objet mathématique n'est pas bien défini. On en discute dans cet exposé, en expliquant pourquoi La singularité n'est pas forcément dangereuse en tout . Qu'au contraire les Singularités sont ces petites choses qui en entrainent de grandes. Qui dessinent le squelette d'un système. Pour ce faire, on fait un parallèle entre Poincaré et Van Gogh, en insistant sur la tétratypologie de Poincaré (sa classification des points singuliers d'un système dynamique) qu'il géométrisa en mettant en place des objets mathématiques , illustrés à son insu par Van Gogh (dans sa célèbre peinture : "La nuit etoilee", 1889), objets qui d'emblée attirent le regard : les singularités du système.

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Center manifold and stability in critical cases for some partial functional differential equations

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Abstract: In this work, we prove the existence of a center manifold for some partial functional differential equations, whose linear part is not necessarily densely defined but satisfies the Hille-Yosida condition. The attractiveness of the center manifold is also shown when the unstable space is reduced to zero. We prove that the flow on the center manifold is completely determined by an ordinary differential equation in a finite dimensional space. In some critical cases, when the exponential stability is not possible, we prove that the uniform asymptotic stability of the equilibrium is completely determined by the uniform asymptotic stability of the reduced system on the center manifold.

Keywords: Hille-Yosida operator, integral solution, semigroup, variation of constants formula, center manifold, attractiveness, reduced system, critical case, asymptotic stability, approximation
2020 Mathematics Subject Classification:

References

2nde International Workshop on Advance in Nonlinear Dynamical Systems, Complex Networks and Applications (DySyX'2025)
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Asymptotic behavior of an epidemic model with infinitely many variants

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Abstract: We investigate the long-time dynamics of a SIR epidemic model with infinitely many pathogen variants infecting a homogeneous host population. We show that the basic reproduction number R0 of the pathogen can be defined in that case and corresponds to a threshold between the persistence ($R_0 > 1$) and the extinction ($R_0 \leq 1$) of the pathogen. When ($R_0 > 1$) and the maximal fitness is attained by at least one variant, we show that the systems reaches an equilibrium state that can be explicitly determined from the initial data. When ($R_0 > 1$) but none of the variants attain the maximal fitness, the situation is more intricate. We show that, in general, the pathogen is uniformly persistent and all families of variants that have a uniformly dominated fitness eventually get extinct. We derive a condition under which the total mass of pathogens converges to a limit which can be computed explicitly. We also find counterexamples that show that, when our condition is not met, the total mass of pathogen may converge to an unexpected value, or the system can even reach an eternally transient behaviour where the mass oscillates between several values. We illustrate our results with numerical simulation..

Keywords:
2020 Mathematics Subject Classification: .

References

State-Dependent Delays: A Gateway to Complex Dynamics

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Abstract: In this talk, I will explore how state-dependent delays-where the delay dynamically changes with the system's state-unlock remarkably rich and complex behaviors. Even in simple models, such delays can produce oscillations, chaos, and other complex behaviors over time. I will present examples, simulations, and key ideas to illustrate the sensitivity and nonlinear effects inherent in these systems. The goal is to give an accessible introduction to a topic that connects delay equations with complex dynamics.

Keywords:
2020 Mathematics Subject Classification: .

References

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Impact of Water Infiltration on the Emergence of Banded Vegetation in Semi-Arid Hillside Regions.

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Abstract: Vegetation patterns, such as stripes, spots, and mixed types, have been observed in arid and semi-arid regions. In this study, we investigate a vegetation-water model on semi-arid hill-slopes, utilizing a reaction-diffusion system to explore the formation of banded vegetation patterns. Conditions for the occurrence of a Hopf bifurcation in the ordinary differential equation (ODE) are established, and we derive the criteria that determine the direction of the bifurcation and the stability of the bifurcated periodic solutions at the nontrivial equilibrium. Subsequently, for the reaction-diffusion equation (RDE), we identify the Turing instability region. To analyze the system's behaviour near the Turing instability point, a weakly nonlinear analysis is employed to derive amplitude equations. Finally, the theoretical findings are validated through numerical simulations, and the environmental factor responsible for the appearance of the banded vegetation pattern is revealed.

Keywords: Vegetation patterns, Hopf bifurcation, Turing analysis, Amplitude equations.

Calabi-Yau Dynamical System and Black Hole Physics

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Abstract: Combining toric geometry and $\mathcal{N} = 2$ supergravity formalisms, we study 5D black branes in the M-theory compactification on a four parameter Calabi-Yau threefold. First, we investigate 5D BPS and non-BPS black holes that are derived by wrapping M2-branes on non-holomorphic 2-cycles in such a toric Calabi-Yau manifold. Concretely, we provide the allowed electric charge regions of BPS and non-BPS black hole states that are obtained by surrounding M2-branes over appropriate 2-cycles. Then, we approach the black hole thermodynamic behavior by computing the entropy and the temperature. By evaluating the recombination factor, we examine the stability of such non-BPS black holes. Precisely, we find stable and unstable solutions depending on the allowed electric charge regions. After that, we study 5D black strings by wrapping M5-branes on non-holomorphic dual 4-cycles in the proposed toric Calabi-Yau manifold by focusing on the stability behaviors. In the allowed regions of the moduli space of the non-BPS stringy solutions, we find stable and unstable states depending on the magnetic charge values.

Keywords: 5D $N = 2$ supergravity formalism, Black holes, Black strings, Calabi-Yau manifolds, Stability Behaviors.

References

[1] A.belhaj and **A.bouhouch**. *Behaviors of black holes and black strings in M-theory on Calabi–Yau manifolds*, *International Journal of Geometric Methods in Modern Physics*, doi: 10.1142/S021988782450169X.

[2] A.belhaj and **A.bouhouch**. On stability behaviors of 5D M-theory black objects. *Communications in Theoretical Physics,*, *arXiv: arXiv: 2403.16724v1 25 Mar 2024.*

[3] A.belhaj, H.belmahi, **A.bouhouch** and SE Ennadifi. On 5D Black Brane Stabilities from M-theory on Three Parameter Calabi-Yau Threefolds, *International Journal of Modern Physics A*, arXiv: 2405.15937, 2024.

[4] A.belhaj, H.belmahi, **A.bouhouch**, SE Ennadifi and M.B.Sedra. Black Holes and Black Strings in M-theory on Calabi-Yau threefolds with four Kähler parameters, arXiv:2501.07167v1 [hep-th] 13 Jan 2025.

Reflections on the relationship between chaos and randomness

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Abstract: This article examines the subtle relationship between chaos and randomness, two concepts that, although they refer to seemingly unpredictable phenomena, are based on fundamentally different principles. Chaos manifests in deterministic sys- tems where small variations in initial conditions lead to unpredictable long-term be- haviors, while randomness pertains to intrinsically probabilistic processes, character- ized by fundamental uncertainty. Although these phenomena are based on distinct mechanisms, they can interact and converge in contexts as varied as the modeling of natural phenomena, climate forecasts, or financial markets. Despite their differences, these two phenomena share common characteristics, such as the absence of apparent order and an unpredictability that defies our attempts at long-term prediction. Through an analysis of chaos theory and probability, this article aims to clarify the distinctions and highlight the deep connections between these two concepts in real systems. The objective of this work is to propose an in-depth reflection on how deterministic chaos and probabilistic randomness intersect, complement each other, and mutually influence in complex systems.

Keywords: Chaos; Randomness; Deterministic systems; Probabilistic processes; Unpredictability; Chaos theory; Probability theory; Complex systems; Chaotic and random interactions

2020 Mathematics Subject Classification. 37H05, 37H12, 60A10.

References

[1] Birkhoff, G. D. (1931). *Proof of the Ergodic Theorem.* Proceedings of the National Academy of Sciences, 17(12), 656–660. https://doi.org/10.1073/pnas.17.12.656

[2] Devaney, R. L. (1989). *An Introduction to Chaotic Dynamical Systems (2nd ed.).* Addison-Wesley.

[3] Feigenbaum, M. J. (1978). *Quantitative Universality for a Class of Nonlinear Transformations.* Journal of Statistical Physics, 19(1), 25–52

[4] Henon, M. (1976). *A two-dimensional mapping with a strange attractor.* Communications in Mathematical Physics, 50(1), 69–77.

[5] Yuri V. Prokhorov. *Convergence of random processes and limit theorems in probability theory.* Theory of Probability and its Applications, 1(2):157–214, 1956. DOI: 10.1137/1101016.

[6] Wirsing, E. (1950). *The Gauss-Kuzmin-Wirsing Distribution.* Mathematics of Computation, 4(36), 142-157.

MODELLİNG THE IMPACT OF EDUCATİON AND MEMORY ON THE MANAGEMENT OF DİABETES MELLİTUS USİNG ATANGANA-BALEANU-CAPUTO FRACTİONAL ORDER MODEL

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Abstract: In this research, the Atangana-Baleanu-Caputo (ABC) fractional derivative is applied to study the memory-dependent behaviors of human awareness regarding a healthy lifestyle and genetic knowledge of diabetes transmission using a proposed mathematical model. Through theorems and proofs, we establish the positivity of solutions and define invariant regions. By employing the Lipschitz and fixed-point theories, we demonstrate the existence of a unique solution for the fractional-order model. The models equations, expressed as an infinite series using the Laplace Adomian Decomposition Method (LADM), are used for numerical experiments. These experiments provide valuable insights, showing that proactive family education significantly reduces prediabetes incidence among genetically predisposed individuals, while increasing awareness enhances memory effectiveness against harmful lifestyle choices, leading to a rapid decline in diabetes cases.

Keywords: Atagana Baleanu Caputo fractional derivative, Diabetes mellitus, Laplace-Adomian decomposition method
2020 Mathematics Subject Classification: Primary 92B05.

References

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Stability and Hopf Bifurcation in a Virotherapy Model with Time Delay

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Abstract: In this paper, we propose a delayed mathematical model which models the virotherapy treatment of a cancer dynamics with logistic growth. The total cancer population is divided into two types of cells: uninfected and infected cells. Considering time delay as a parameter, we establish the positivity and boundedness of solutions and the stability of the possible equilibria. We prove that, time delay can lead to periodic oscillations with small amplitude called "Jeff's phenomenon" which observed in laboratory and causes oscillations in cancer size via Hopf bifurcation theory. We give an algorithm determining the direction of Hopf bifurcation via center manifold and normal form theories. We end with some numerical simulations illustrating our obtained results

Keywords: Virotherapy, Stability, Hopf bifurcation, Center manifold, Normal form.

References

[1] Komarova, N. L. and Wodarz, D. ODE models for oncolytic virus dynamics, J. Theor. Biol. 263, 2010, 530–543.

[2] Smith J., How to add could. Acta Mathematicae, 268(1), 2021, 12012-12312.

[3] Najm, F., Yafia, R. and Aziz-Alaoui, M. A. ,Hopf Bifurcation in Oncolytic Therapeutic Modeling: Viruses as Anti-Tumor Means with Viral Lytic Cycle, International Journal of Bifurcation and Chaos, 32 (11), 2250171, 2022.

[4] Wodarz, D. and Komarova, N. , Computational biology of cancer: Lecture notes and mathematical modeling, World Scientific, 2008

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Numerical Methods for Solving the One-Dimensional Birkhoff Polynomial Interpolation Problem

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Abstract: The subject of this Presentation aims to study the Birkhoff polynomial interpolation [3]. Given a set $Z = \{z_0, ..., z_n\}$ with $(n+1)$ nodes, of ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$), we will seek to study questions of existence and of uniqueness [6, 4, 2] of a polynomial P such that P and a number of its derivatives take, at these nodes, given values, as well as its representation [1, 5]. The algorithmic aspect will also be taken into consideration, and a numerical implementation of the obtained solutions must be carried out. Complexity studies algorithmic, numerical stability and comparisons with other solutions are expected from this work.

Keywords: Birkhoff interpolation - lacunar polynomial - Schur complement - Recursive algorithm
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] O. Rhouni, M. Errachid, M. Esghir, ***Different Approaches for Solving the One-Dimensional Birkhoff Polynomial Interpolation Problem*** ; Numerical Algorithms Journal, (2025), (soumis).

[2] K. Atkinson, A. Sharma, ***A Partial Characterization of Poised Hermite-Birkhoff Interpolation Problems*** ; SIAM Journal on Numerical Analysis, 6 (1969), 230–235. https://epubs.siam.org/doi/abs/10.1137/0706021

[3] G. D. Birkhoff, ***General Mean Value and Remainder Theorems with Applications to Mechanical Differentiation and Quadrature*** ; Transactions of the American Mathematical Society, 7 (1906), 107–136. https://www.jstor.org/stable/1986339

[4] D. Ferguson, ***The question of uniqueness for G. D. Birkhoff interpolation problems*** ; Journal of Approximation Theory, 2 (1969), 1–28. https://www.sciencedirect.com/science/article/pii/0021904569900288

[5] A. Messaoudi, M. Errachid, K. Jbilou, H. Sadok, ***GRPIA: a new algorithm for computing interpolation polynomials*** ; Numerical Algorithms Journal, 80 (2019), 253–278. https://doi.org/10.1007/s11075-018-0543-x

[6] G. Pólya, ***Bemerkung zur Interpolation und zur Näherungstheorie der Balkenbiegung*** , ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik, 11 (1931), 445–449. https://onlinelibrary.wiley.com/doi/abs/10.1002/zamm.19310110620

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Doubly Reflected BSDEs for the Valuation of Dynkin Games in Defaultable Settings

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Abstract: Default risk is one of the most extensively studied aspects of credit risk, attracting significant attention in the last decades. This presentation addresses the valuation problem for Dynkin games with a fixed expiry time in a defaultable environment, specifically focusing on American game options. The information flow $\mathbb{G} := (\mathcal{G}_t)_{t\geq 0}$ is driven by a standard Brownian motion W in the filtration $\mathbb{F} := (\mathcal{F}_t)_{t\geq 0} = (\sigma\{W_s : s \in [0,t]\})_{t\geq 0}$, along with a random time $\tau : \Omega \to (0, +\infty)$, known as the *default time*, which is not a stopping time with respect to $\mathbb{F}$. Our approach uses the theory of Doubly Reflected Backward Stochastic Differential Equations (DRBSDEs) with two distinct right-continuous with left-limits (RCLL) barriers.
Our methodology involves two approaches:

1. In the first approach, we assume the immersion property holds and solve the pricing problem of an American game option in a financial market with default using DRBSDEs with a stochastic Lipschitz coefficient.
2. In the second approach, we relax the $\mathcal{H}$-hypothesis for the stopped process W^τ, applying a change of probability measure.

In both cases, the cost function or the game's upper and lower values are expressed in terms of a non-linear expectation derived from a classical BSDE.

Keywords: Doubly reflected BSDEs, RCLL barriers, Stochastic Lipschitz coefficients, Progressive enlargement of filtration, Game options, Default time, Non-linear expectations.
2020 Mathematics Subject Classification: Primary 60H30 , 60H15, 60H20.

A Dynamical model for the epidemiological interaction between COVID-19 and Tuberculosis

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Abstract: This study examines a mathematical model that captures the transmission dynamics of COVID-19 and tuberculosis within a population, emphasizing the interplay between the two diseases. The model incorporates key epidemiological factors, such as co-infection and disease progression, to better understand their joint impact. The existence and uniqueness of the endemic equilibrium are rigorously proven, followed by a comprehensive analysis of the equilibrium points and their local stability. To provide deeper insights, a sensitivity analysis is conducted to identify critical parameters influencing disease dynamics. Finally, numerical simulations are carried out to validate the theoretical results and explore potential scenarios, offering valuable perspectives for public health strategies.

Keywords: COVID-19, Tuberculosis, Co-infection, Reproduction number.
2020 Mathematics Subject Classification: 92D30, 34A34, 34D23, 34C23 .

References

[1] M. Diagne, H. Rwezaura, S. Tchoumi, J. Tchuenche, A mathematical model of COVID-19 with vaccination and treatment, Comput.Math. Methods Med. 2021 (2021).

[2] M. Kumar, D. Surendran, M. Manu, P. Rakesh, S. Balakrishnan, Mortality of TB-COVID-19 coinfection in India, Int J.TB Lung Dis.(2021)250–251.

[3] P. Van den Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Math. Biosci. 180 (1–2) (2002) 29–48.

On stochastic Gilpin Ayala population model with Markovian switching

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Abstract: In this paper, we analyze a stochastic Gilpin Ayala population model with Markovian switching and white noise. The Gilpin Ayala parameter is also allowed to switch. We establish the global stability of the trivial equilibrium state of the model. Verifiable sufficient conditions which guarantee the extinction and persistence are provided. Furthermore, we show the existence of a stationary distribution. The analytical results are illustrated by computer simulations.

Keywords: Markovian switching; Stability; Persistence; Stationary distribution.

References

[1] A. Settati, A.Lahrouz. *Stationary distribution of stochastic population systems under regime switching* .Appl. Math. Comput, **(244)**: 235–243, 2014.

[2] M. Vasilova, M. Jovanovic. Stochastic Gilpin–Ayala competition model with infinite delay. Math. Comput, **(217),**: 4944–4959, 2010.

[3] P. Xia, X. Zheng., D. Jiang.Persistence and non-persistence of a nonautonomous stochastic mutualism system. Abstr. Appl. Anal, **(217),**:2013.

[4] C. Zhu, G. Yin. Asymptotic properties of hybrid diffusion systems. J. Control Optim, **46**, 1155–1179: 2007.

A Comprehensive Seismic Vulnerability Assessment of Architecturally Irregular Buildings in Al Hoceïma, Northern Morocco: An Analysis of Structural Risks and Resilience

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Abstract: Participants are encouraged to enroll and submit one page extended abstract written in English, in accordance with the DySyX'2025 Latex template.

Keywords: The assessment of seismic vulnerability is crucial to determine the performance of existing buildings during earthquakes [1]. Northern Morocco has recently been struck by several devastating earthquakes, resulting in casualties and significant losses, highlighting the importance of considering seismic risk, particularly the vulnerability of existing buildings in this region [2]. Al Hoceima, located in a seismically active zone with slopes ranging from 10deformation generates seismic activity that can sometimes have catastrophic consequences [4]. Seismological studies indicate that the main shock occurred on land, at the boundary between the external and internal Rif zones, at a depth of 10 to 14 km . The aim of this study is to deepen the understanding of the seismic vulnerability of irregular structures subjected to seismic loading.
Keywords:: seismic, Irregular structure, vulnerability

References

[1] Ganesh. G, Sanjay. B, Seismic performance of re-entrant corner building under the different earthquake direction. Asian.J. Converg. Technol, 4 (3) (2018) 5-11. . Editor: Contry, year.

[2] Siva. N, Nimmy Mariam . A, Anitha . K. Analysis of irregular structures under earthquake loads. in Procedia Structural Integrity. (2019), 806-819..

[3] Taj-Eddine. C, Ahmed E. H. (2012): Seismicity and Seismic Hazard in Morocco : 1901-2010. Bulletin de l'Institut Scientifique, Rabat, section Sciences de la Terre, 2012, n* 34, p. 45-55. .

[4] Hatzfeld, D., Caillot, V., Cherkaoui, T.E., Jebli, H., Medina, F., 1993. Microearthquake seismicity and fault plane solutions around the Nekor strike-fault, Morocco. Earth Planet. Sci. Lett. 120, 31-34..

2-$\mathcal{K}$-frames in 2-Hilbert spaces

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Abstract: The aim of my article, we introduce a 2-$\mathcal{K}$-frames in the 2-Hilbert space $\mathcal{H}$, where $\mathcal{K}$ is a θ-bounded linear operator on $\mathcal{H}$ for a fixed element θ in $\mathcal{H}$, and explore some properties of them. Also, we will characterize the 2-$\mathcal{K}$-frames by the 2 pre-$\mathcal{K}$-frame operators, the 2-$\mathcal{K}$-frame operators, likewise to what is seen in the case of Hilbert spaces. In the rest of the article, we will set a definition of θ-atomic systems for $\mathcal{K}$, and give some results concerning this notion. Finally we will find out a representation of each element in a closed range $R(\mathcal{K})$.

Keywords: 2-$\mathcal{K}$-frame, 2-$\mathcal{K}$-frame operator, 2-Hilbert space.
2020 Mathematics Subject Classification: Functional analysis 46-XX.

References

[1] M. Christensen, Ole. *Frames and bases: An introductory course*.publisher=Springer Science & Business Media : Maryland, 2008.

[2] F. Harikrishnan, PK and Riyas, P and Ravindran, KT. Riesz theorems in 2-inner product spaces. *Novi Sad Journal of Mathematics*, **41** (2): 57–61, 2011.

[3] F. Xiao, Xiangchun and Zhu, Yucan and Găvruţa, Laura. Some properties of K-frames in Hilbert spaces. *Results in mathematics*, **63** (3): 1243–1255, 2013.

On the Spectral Decomposition for Partial Functional Differential Equations

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Abstract: The aim of this work is to study the essential growth rate of the semigroup generated by linear partial functional differential equations in the phase space. This investigation considers cases where the semigroup associated with the non-delay part is neither necessarily compact nor uniformly exponentially stable. Applications include studying the behavior near hyperbolic equilibrium points in nonlinear cases.

Keywords: Partial functional differential equations, Semigroup, Essential spectral radius, Invariant manifolds
2020 Mathematics Subject Classification: Primary 34k30, 34K19, 34K20.

References

[1] R. Benkhalti, K. Ezzinbi and S. Fatajou, Stable and unstable manifolds for nonlinear partial neutral functional differential equations, Differential Integral Equations 23 (2010), no. 7-8, 773–794.

[2] N. Drisi and B. Es-sebbar, A Bohr-Neugebauer property for abstract almost periodic evolution equations in Banach spaces: Application to a size-structured population model, J. Math. Anal. Appl. 456 (2017), no. 1, 412– 428.

[3] A. Ducrot, Z. Liu, P. Magal, Essential growth rate for bounded linear perturbation of non densely defined Cauchy problems, J. Math. Anal. Appl. 341 (2008) 501–518.

[4] H. R. Henriquez, On non-exact controllable systems. Int. J. Control 42, 71-83 (1985).

[5] J. Wu, Theory and Applications of Partial Functional Differential Equations," Graduate Texts in Applied Mathematics, Springer-Verlag, Vol. 119, (1996).

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Deep learning and optimal control methods for fractional epidemiological model

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Abstract: The main goal of this work is to address fractional forward and inverse problems using a deep learning method. A fractional Physics-Informed Neural Network (fPINN) is proposed to solve Fractional Differential Equations (FDE) in epidemiology. The model minimizes the combined mean-squared residuals of FDE and the mean-squared errors in the initial and boundary conditions to fit the observed data. This study presents a novel method for estimating fractional-order derivatives within a mathematical epidemic model that includes fractional boundary conditions, utilizing real-world data for both training and validation.

Keywords: Fractional Physics-Informed Neural Networks, Fractional Differential Equations, Optimal control, Epidemic model.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] H. Aghdaoui, A. A. Raezah, M. Tilioua, Y. Sabbar. Exploring COVID-19 model with general fractional derivatives: novel Physics-Informed-Neural-Networks approach for dynamics and order estimation. *J. Math. Comput.*, Sci,Vol. pp. 142-162, 2025.

[2] J. Hou, Z. Ma, S. Ying, & Y. Li, HNS: An efficient hermite neural solver for solving time-fractional partial differential equations. *Chaos, Solitons & Fractals,* 181, 114637, 2024

Modélisation mathématique de la dynamique du cancer en réponse à l'immunothérapie et à la virothérapie : Approches déterministes et stochastiques

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Abstract: Cette étude présente une analyse mathématique de la dynamique du cancer à l'aide de modèle déterministe et stochastique, en se concentrant sur les interactions entre les cellules tumorales, les cellules effectrices immunitaires et les virus oncolytiques. Nous développons un modèle basé sur des équations différentielles ordinaires (EDO) et des équations différentielles stochastiques (EDS) pour capturer ces interactions. Notre analyse commence par un examen de la bonne pose du modèle, suivie d'une étude détaillée de la stabilité de l'équilibre, y compris les conditions d'extinction et de persistance du cancer. Enfin, nous validons nos résultats théoriques par des simulations numériques, fournissant des informations sur les réponses dynamiques du cancer sous immunothérapie et virothérapie.

Keywords: Modélisation du cancer, EDO, EDS, stabilité, simulation numérique.
2020 Mathematics Subject Classification: 2010 MSC : 92C50, 60Hxx, 34Dxx, 65-XX.

References

[1] D. Wodarz and N. Komarova. Dynamics of Cancer: Mathematical Foundations of Oncology. *World Scientific*, **2014**.

[2] F. Nani and H. I. Freedman. A mathematical model of cancer treatment by immunotherapy. *Mathematical Biosciences*, **163** (2): 159–199, 2000.

[3] M. Elkaf, A. Meskaf, and K. Allali. Dynamics of cancer cells with immunotherapy and virotherapy. *Commun. Math. Biol. Neurosci.*, **2021**: Article–ID, 2021.

Quantitative study for some partial differential inclusions with memory

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Abstract: This study aims to investigate the existence of mild solutions for certain partial integrodifferential inclusions under nonresonance conditions. The primary tools employed are the theory of resolvent operators and a fixed point theorem of Ma's type[1]. An example is provided at the end to illustrate the abstract results.

Keywords: integrodifferential inclusions, mild solution, resolvent operator.
2020 Mathematics Subject Classification: Primary 47H05, 35B15.

References

[1] T. W. Ma. Topological degrees of set-valued compact fields in locally convex spaces. *Dissertationes Mathematicae*, **92**, 1972.

Optimal control strategies for a fractional leprosy model with environmental bacterial load

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Abstract: In this study, we introduce a mathematical model to analyze the dynamics of Leprosy transmission, incorporating a novel SEIR framework extended by an additional compartment that accounts for the bacterial load in the environment. The model uses the Caputo-Fabrizio (CF) fractional derivative to improve the representation of memory effects in the disease transmission process. We have proven the existence and uniqueness of the solution using the Banach fixed-point theorem, along with the local and global stability of the equilibrium steady states. A comprehensive sensitivity analysis is conducted to identify the key parameters influencing leprosy spread. Numerical simulations are performed to demonstrate the model's capability to capture the complex dynamics of leprosy transmission. Additionally, an optimal control strategy is proposed, utilizing two control variables: raising awareness and administering medical treatment to reduce the number of infected individuals. The findings provide valuable insights into the effective management of leprosy, pwointing out the critical role of both environmental factors and public health interventions.

Keywords: Leprosy virus, Caputo-Fabrizizo derivative, Fractional calculus, bacterial load.
2020 Mathematics Subject Classification: Primary 92D30, 26A33, 34A08, 92C50.

References

[1] S. Kumar, B. Bhushan, H. K. Kar, S. Dogra. IAL textbook of leprosy. *Jaypee Brothers Medical Publishers*, **2023**.

[2] C. A. Roberts. Leprosy: Past and present. *University Press of Florida*, **2020**.

[3] W. H. Organization, O. Mondiale de la Santé, et al. Global leprosy (Hansen disease) update, 2022: new paradigm—control to elimination—situation de la lèpre (maladie de Hansen) dans le monde, 2022: nouveau paradigme—de la lutte contre la maladie à son élimination. *Wkly. Epidemiol. Rec.*, **98**(35): 409–429, 2023.

Existence of solutions for some nonlocal elliptic problems

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Abstract: The main objective is to prove the existence of weak solutions to a class of Kirchhoff problems of the type:

$$-M\Big(\int_\Omega A(x,\nabla u)dx\Big)\operatorname{div} a(x,\nabla u) = f(x,u) \quad \text{in } \Omega.$$

The functions A and a are supposed to satisfying some conditions which will be used through the approximate Gelerkin method and tools from Young measures. We acheive the existence of weak solutions under classical growth conditions of Leray-Lions type.

Keywords: Quasilinear elliptic systems, nonlocal problems, Galerkin approximation
2020 Mathematics Subject Classification: Primary 35J60, 35J25, 35D30, 28Axx.

References

[1] E. Azroul and F. Balaadich (2021), Existence of Solutions for a Class of Kirchhoff-Type Equation via Young Measures, *Numerical Functional Analysis and Optimization*, volume **4**, No 42, pp. 460-473.

[2] N. Hungerbühler (1999), Quasilinear elliptic systems in divergence form with weak monotonicity, *New York Journal of Mathematics*,volume **5**, pp 83-90.

[3] L.C. Evans (1990), Weak convergence methods for nonlinear partial differential equations, *American Mathematical Soc.*, Number **74**.

On the fractional Musielak Sobolev spaces with variable s(.,.)-Order : Results and Applications

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Abstract: In the last years, great attention has been devoted to the study of nonlinear problems involving nonlocal operators in modular spaces. In particular, in the fractional Musielak Sobolev spaces. In this talk, we introduce the fractional MusielakSobolev spaces with variable order. Also, we show that there exists an eigenvalue for the nonlocal problem, by means of Ekeland's ariational principle.

Keywords: Fractional Musielak Sobolev spaces, Eigenvalue problems, Ekeland's Variational principle
2020 Mathematics Subject Classification: 35R11 .

References

[1] E. Azroul , A. Benkirane, M. Shimi and M. Srati (2020): On a class of nonlocal problems in new fractional Musielak-Sobolev spaces, Applicable Analysis, DOI: 10.1080/00036811.2020.1789601.

[2] E. Azroul , A. Benkirane , M. Shimi and M. Srati (2020): Embedding and extension results in fractional Musielak-Sobolev spaces, Applicable Analysis, Applicable Analysis, DOI: 10.1080/00036811.2021.1948019.

[3] Srati, M. Eigenvalue type problem in $s(.,.)$-fractional Musielak-Sobolev spaces. J Elliptic Parabol Equ 10, 387-413 (2024). https://doi.org/10.1007/s41808-024-00269-5.

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On the stochastic maximal L^p-regularity for perturbed evolution equations

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Abstract: The concept of stochastic maximal L^p-regularity is a technique utilized to solve non-linear, quasi-linear, as well as non-autonomous linear stochastic evolution equations, driven by a cylindrical brownian motion, in Banach spaces. In [5, 6] sufficient conditions for stochastic maximal L^p-regularity were derived, and various applications to stochastic partial differential equations are worked out. The stochastic maximal L^p-regularity has undergone further development in the work of [1, 4]. The authors have demonstrated that various characteristics of deterministic maximal L^p-regularity can also be applied to the stochastic version. Additionally, a significant contribution is made in Section 6 of [1], where the authors prove the invariance of stochastic maximal L^p-regularity under a specific category of unbounded linear perturbations. In this talk, we aim to provide further insights into the stability of stochastic maximal L^p-regularity when subjected to perturbation. Specifically, Miyadera-Voigt perturbations. These results, see [3], generalize those obtained in the deterministic case, see e.g. [2] and complement the ones presented in [1].

Keywords: Stochastic equations, maximal regularity, perturbation, admissible observation operators.
2020 Mathematics Subject Classification: Primary 60H15; Secondary 35B65; 47D06.

References

[1] A. Agresti and M. Veraar, Stability of stochastic maximal L^p-regularity, J. Math. Anal. Appli 482, 123553 (2020).

[2] Amansag, A., Bounit, H., Driouich, D., Hadd, S.: On the maximal regularity for perturbed autonomous and non-autonomous evolution equations. J. Evol. Equ. 20, 165–190 (2020)

[3] Bounacer, H., Hadd, S.: Stability of stochastic maximal L^p-regularity under admissible observation operators. Arch. Math. (2025). https://doi.org/10.1007/s00013-024-02096-5

[4] Lorist, E. and Veraar, M.: Singular stochastic integral operators. Analysis & PDE (2019). **482**, 123553 (2020).

[5] Van Neerven, J. M. A. M., Veraar, M. C. and Weis, L.: Maximal L^p-regularity for stochastic evolution equations, SIAM J. Math. Anal. 44 (3) (2012), 1372-1414.

[6] Van Neerven, J. M. A. M., Veraar, M. C. and Weis, L.: Stochastic maximal L^p-regularity. Ann. Probab. 40(2) (2012) 788-812.

Optimal Control of a Reaction-Diffusion Problem: SIR Epidemiological Model

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Abstract: This work addresses an optimal control problem applied to a reaction-diffusion SIR (Susceptible–Infected–Recovered) model to design an effective vaccination strategy against influenza epidemics. The control problem incorporates realistic constraints, including a limited total vaccination coverage and a maximum daily vaccine administration capacity, modeled as inequality constraints on the state variables. We demonstrate the existence of an optimal control solution and analyze an optimality system using a penalty function to handle the associated constraints. A gradient-based algorithm is proposed to solve this system. The spatial SIR model is solved using the finite difference method (FDM) for the temporal dimension and the finite element method (FEM) for the spatial dimension. Numerical simulations reveal that the optimal vaccination strategy varies across regions, depending on the disease propagation rate.

Keywords: SIR epidemic , equation reaction diffusion , optimal controle
2020 Mathematics Subject Classification: Primary, 49M27, 49M29.

References

[1] J. Jang, Hee-Dae Kwon, and J. Lee. Optimal control problem of an sir reactiondiffusion model with inequality constraints. Mathematics and Computers in Simulation, 171:136-151,2020.

[2] B.M. Adams, H.T. Banks, M. Davidian, Hee-Dae Kwon, H.T. Tran, S.N. Wynne, E.S. Rosenberg, HIV dynamics : Modeling, data analysis, and optimal treatment protocols, J. Comput. Appl. Math. 184(2005)10-49.

[3] L.J.S. Allen, B.M. Bolker, Y. Lou, A.L. Nevai, Asymptotic profiles of the steady states for an SIS epidemic reaction-diffusion model, Discrete Contin. Dyn. Syst.21(2008) 1-20.

Unveiling memory-driven dynamics: Fractional calculus and microswimmer trajectories in 1D shear flow

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Abstract: This study investigates the captivating dynamics of spherical microswimmers in a 1D shear flow, revealing the intricate interplay between memory effects and fluid mechanics through the lens of fractional calculus. By deriving exact solutions for the orientation and trajectory of these microswimmers, we uncover a rich tapestry of motion patterns that challenge traditional models. Our findings not only enhance the theoretical understanding of microswimmer behavior but also hold significant implications for practical applications in biophysics and targeted drug delivery. This research underscores the power of fractional calculus as a transformative tool in unraveling the complexities of biological systems, paving the way for innovative approaches in the study of fluid dynamics.

Keywords: Fractional calculus, The modified Riemann–Liouville fractional derivative, Mittag-Leffler function, Spherical microswimmer, Exact trajectory, Fractional differential equations, Memory effect.
2020 Mathematics Subject Classification: 26A33, 34A08, 76Z99, 92C10.

References

[1] M. Guedda, J. Chaiboub, M. Benlahsen, C. Misbah, Exact trajectory solutions of a spherical microswimmer under flow and external fields, Phys. Rev. Fluids 6 (7) (2021) 074102.

[2] G. Jumarie, Modified Riemann-Liouville derivative and fractional Taylor series of nondifferentiable functions further results, Comput. Math. Appl. 51 (9–10) (2006) 1367–1376.

[3] E. Azroul, G. Diki, M. Guedda, Exploring the interplay between memory effects and vesicle dynamics: A five-dimensional analysis using rigid sphere models and mapping techniques, Stud. Appl. Math. 152 (1) (2024) 431–452.

[4] E. Azroul, G. Diki, Analytical investigation of vesicle dynamics via the modified Riemann–Liouville fractional derivative: Mittag-Leffler function solution and comparative analysis with Caputo's derivative, Chaos 34 (6) (2024).

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Fractional-order modeling of parasite-produced marine diseases with memory effect

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Abstract: This paper investigates the mathematical modeling and dynamics of marine diseases caused by parasites, incorporating memory effects. A fractional-order Susceptible-Infectious-Parasite (SIP) model is proposed using the Caputo fractional derivative. The basic reproduction number is computed, and the positivity, boundedness, existence, and uniqueness of solutions are analyzed. Additionally, the local stability of equilibrium states is studied. Finally, numerical simulations validate the theoretical findings.

Keywords: Fractional derivative, Caputo derivative, Caputo derivative, parasite-induced marine diseases, memory effect, equilibrium points, stability analysis
2020 Mathematics Subject Classification: 26A33, 34A08, 92D30, 37N25.

References

[1] I. Podlubny. Fractional Differential Equations (1998, Academic Press)

[2] M. Caputo. Linear models of dissipation whose q is almost frequency independent—II. Geophysical Journal International, 13 (5): 529–539, 1967.

[3] A. Alla Hamou, E. Azroul, and S. L'kima. Fractional-order modeling of parasite-produced marine diseases with memory effect. Modeling Earth Systems and Environment, 10 (5): 6357–6372, 2024.

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Electromagnetic energy harvesting from self-excited oscillations

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Abstract: This study investigates vibration-based electromagnetic energy harvesting using an excited van der Pol oscillator coupled to a storage circuit incorporating via electromechanical coupling mechanism interaction. The modulation equations governing the vibration and output voltage responses are derived using the averaging method, and analytical approximate solutions for the periodic responses are obtained. By applying a second-step perturbation method to the slow flow, approximate quasiperiodic solutions for the system's response and the associated output voltage are derived. The energy harvesting performance of the system is evaluated from multiple frequency sources simultaneously exists in the system. The influence of various harvester parameters on the output voltage is analyzed in both the periodic and quasiperiodic regimes. The advantages of employing a coupled van der Pol oscillator with a circuit for energy harvesting are illustrated in both periodic and quasiperiodic regimes. This approach enhances energy harvesting optimization by taking advantage of the dynamics exhibited by these oscillators, which can exhibit complex behaviors such as self-excitation and bifurcations. Studies indicate that the interaction between these oscillators enhances the efficiency of energy harvesting systems, particularly when considering various excitation types and damping models. The ability to operate effectively in both periodic and quasiperiodic states allows for more robust energy harvesting solutions across different operational conditions. Numerical simulations are conducted to validate and support the analytical results.

Keywords: energy harvesting, van der Pol oscillator, periodic and quasiperiodic vibrations, power

2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] N. Yu, C. Wu, G. Yu, B. Yan. Bistable electromagnetic energy harvesting enhanced with a resonant circuit. NODYCON Conference Proceedings Series. 221-231. 2022.

[2] P. Soltani, G. Kerschen. The nonlinear piezoelectric tuned vibration absorber. *Smart Mater. Struct.* **24**, 075015. 2015

[3] J. Guckenheimer, P. Holmes. *Nonlinear Oscillation and Bifurcation of Vector Fields.* Springer, New York. 1993

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Fractional PDE Modeling of an Epidemiological System: Optimal Control and Spatial Distribution

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Abstract: In this study, we investigate a fractional reaction-diffusion SEIR epidemiological model using the Caputo-Fabrizio fractional derivative with a non-singular kernel and the Laplacian operator. Our objective is to analyze infectious disease transmission while considering spatial heterogeneity effects. The government's vaccination program is incorporated as a control variable within the fractional partial differential equation model.

We establish the existence and uniqueness of a positive solution by leveraging results from Laplacian operator theory and fractional Laplace transforms, providing a foundation for optimal control analysis. Our primary goal is to determine the optimal control strategy that minimizes both the number of infections and the associated vaccination and treatment costs over a finite time and spatial domain. Employing minimizing sequences and weak convergence results, we rigorously prove the existence of an optimal control and characterize it through state and adjoint functions.

To illustrate the practical impact of our findings, we conduct numerical simulations using the Forward-Backward Sweep method. Results reveal that for integer-order derivatives, disease spread is more rapid, whereas for fractional-order cases, the infection propagates more gradually while maintaining similar vaccination costs. Finally, our analysis confirms that an effective vaccination strategy significantly reduces disease transmission and enhances epidemic control.

Keywords: Epidemiological Model; Factional Derivatives; Simulations; Optimal Control.
2020 Mathematics Subject Classification: 92C60, 26A33, 33F05, 49J20..

References

[1] R. M. Anderson, R. M. May; *Regulation and stability of host-parasite population interactions*; Journal of animal ecology, 47, 219–247, 1978.

[2] D. Bernoulli, D. Chapelle; *Essai d'une nouvelle analyse de la mortalité causée par la petite vérole, et des avantages de l'inoculation pour la prévenir*; 1–45, 1760.

[3] W. O. Kermack, A. G. McKendrick; *A contribution to the mathematical theory of epidemics*; Proceedings of the Royal Society A, 115, 700–721, 1927.

[4] A. Zinihi, M. R. Sidi Ammi, M. Ehrhardt; *Optimal control of a diffusive epidemiological model involving the Caputo-Fabrizio fractional time-derivative*; arXiv:2403.00364 [math.DS], 2024.

Nonlinear approximation with frames

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Abstract: This talk aims to focuses on the approximation by frames in separable Hilbert spaces. We present an iterative method to approximate a signal x based on a given frame, by defining a recursive sequence that converges to the signal x. We show that this method converges to a signal that minimizes a specific functional involving the frame coefficients of the given frame. The importance of this method is better understood when the frame used has a large condition number, and the classical frame algorithm is unusable.

Keywords: Approximation, frame, frame algorithm.
2020 Mathematics Subject Classification: 42C15

References

[1] P. G. Casazza, The art of frame theory, Taiwanese J. Math., 4 (2000), pp. 129–202.

[2] O, Christensen, An Introduction to Frames and Riesz Bases, 2nd edn. Applied and Numerical Harmonic Analysis. Birkhäuser/Springer, Cham (2016). https://doi.org/10.1007/978-3-319-25613-9

[3] R. J. Duffin, A. C. Schaeffer, A class of nonharmonic Fourier series, Trans. Amer. Math. Soc., 72 (1952), pp. 341–366.

[4] K. H. Grochenig, Acceleration of the frame algorithm, IEEE Trans. Signal processing 12 (1993), 3331-3340.

V-sets and the property (VLD) in Banach spaces

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Abstract: We study the notion of V-sets in Banach spaces and Banach lattices, and we give some characterizations of it in terms of sequences. As an application, we establish new properties of unconditionally converging operators and 1-Schur property in Banach lattices. Next, by introducing the concept of the property (VLD) in Banach spaces, we investigate the Dunford-Pettis completely continuous property of unconditionally converging operator. Finally, we derive the relationships between the property (VLD) and the relatively compact Dunford-Pettis property (resp., the Pelczynski's property (V)), and we deduce some examples of Banach spaces with the property (VLD).

Keywords: Banach lattice, V-set, relatively compact Dunford-Pettis property, unconditionally converging operator.
2020 Mathematics Subject Classification: Primary 46A40, 46B40.

References

[1] C.D. Aliprantis, O. Burkinshaw, *Positive operators*, Reprint of the 1985 original. Springer, Dordrecht, 2006.

[2] J. Howard, Banach spaces on which every unconditionally converging operator is completely continuous, *Commentationes Mathematicae Universitatis Carolinae*, Vol. **16**, No. (4), 745-753, 1975.

[3] A. Retbi, B. El Wahbi, L-Dunford-Pettis property in Banach spaces, *Methods Func. Anal. Topol*, **22**, 387-392, 2016.

Fractional Navier-Stokes Equations: Existence, Uniqueness, and Optimal Control

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Abstract: The Navier-Stokes equations describe the motion of incompressible fluids and play a fundamental role in fluid mechanics, engineering, and applied mathematics. Since their formulation in the 19th century, they have been the subject of extensive mathematical and numerical investigations. The literature on Navier-Stokes equations covers various aspects, including existence, uniqueness, and regularity of solutions in different functional spaces. The classical results of Leray (1934) established the global existence of weak solutions in three dimensions, but the question of their uniqueness and regularity remains one of the most significant open problems in mathematical physics. In recent years, new approaches involving fractional Laplacians, p-Laplacian operators, and stochastic methods have provided deeper insights into turbulence and anomalous diffusion. Furthermore, computational fluid dynamics (CFD) has contributed to understanding complex flow structures through numerical simulations. This work presents a comprehensive review of the theoretical and computational advancements in Navier-Stokes equations, with particular attention to recent developments in fractional and nonlinear extensions.

Keywords: Navier-Stokes equations, weak solutions, turbulence, fractional Laplacian, p-Laplacian, numerical simulations.

References

[1] J. Leray. *Sur le mouvement d'un liquide visqueux emplissant l'espace*. Acta Mathematica, 63:193–248, 1934.

[2] E. Di Nezza, G. Palatucci, and E. Valdinoci. Hitchhiker's guide to the fractional Sobolev spaces. *Bulletin des Sciences Mathématiques*, **136**(5):521–573, 2012.

[3] O. Jarrín and G. Vergara-Hermosilla. An L^p-theory for fractional stationary Navier–Stokes equations. *Journal of Elliptic and Parabolic Equations*, **10**:859–898, 2024.

[4] F. Flandoli. An Introduction to 3D Stochastic Fluid Dynamics. In *SPDE in Hydrodynamic: Recent Progress and Prospects*, pp. 51–150, Springer, 2008.

[5] H. J. Wirz. A critical review of numerical solution of Navier-Stokes equations. In *Progress in Numerical Fluid Dynamics*, pp. 19–32, Springer, 1975.

A Mathematical Model for Epidemic Dynamics with Multiple Vaccines, Age-Structured Strategies, and Parameter Estimation

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Abstract: Mathematical modeling of infectious diseases plays a fundamental role in understanding their behavior and guiding the development of effective public health strategies. This study introduces a detailed compartmental model that includes age-structured vaccination dynamics and multi-stage disease progression. The model devloped in this study is generalization of that analyzed in the works [1, 2], it is composed of differential equations that outline the dynamics among susceptible, exposed, infected, hospitalized, recovered, and deceased populations, along with age-structured vaccination coverage. The study begins by deriving the governing equations and performing a theoretical analysis of the system, which includes identifying disease-free and endemic steady states and calculating the basic reproduction number ($\mathscr{R}_0$). The second part focuses on parameter estimation using real-world epidemiological data. To minimize the differences between the observed data and the model's simulations across compartments, a weighted least squares method is applied. The model is validated with data from Portugal and Italy, showcasing its capability to capture disease transmission dynamics and the impact of age-targeted vaccination strategies. The results emphasize the significance of integrating age-structured vaccination, waning immunity, and multi-stage disease dynamics into epidemic models to formulate effective control strategies. The study also points to the potential for expanding the model to tackle broader public health issues, such as optimizing vaccination policies and evaluating the effects of behavioral changes.

Keywords: Compartmental epidemic model, Age-structured vaccination, Disease dynamics, Weighted least squares, Basic reproduction number, Global stability, Waning immunity
2020 Mathematics Subject Classification: Primary 92B05, 35B35, 93D05, 65N12.

References

[1] A. Ouakka. A. El Azzouzi and Z. Hammouch. Global dynamic behavior of a vaccination–age SVIR model with treatment and general nonlinear incidence rate. *Journal of Computational and Applied Mathematics*, **422**: 114848, 2023.

[2] A. Ouakka. A. El Azzouzi and Z. Hammouch. An SVIQR model with vaccination-age, general nonlinear incidence rate and relapse: Dynamics and simulations. *International Journal of Biomathematics*, **18** (02): 2350092, 2025.

Optimal control strategies for drug abuse: A reaction-diffusion approach to managing the spread of drug use

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Abstract: In this research, we investigate the complex dynamics of drug addiction in the context of spatial spread. We accomplish this by building an example model carefully constructed to divide the population into discrete subpopulations. We derive a system of equations from this conceptual framework, forming our analytical investigation's foundation. Crucially, our inquiry extends beyond mere theoretical speculation, we delve into the practical realm by interrogating the existence and uniqueness of solutions, in addition to carefully considering tactics intended to reduce drug use and damage the harmful networks of distribution using optimal control strategies.

Keywords: Drug abuse, Reaction-diffusion models, optimal control, and compartmental modeling

References

[1] Pierre Magal, Shigui Ruan, et al. *Theory and applications of abstract semilinear Cauchy problems*.volume 201. Springer, 2018.

[2] mnon Pazy. *Semigroups of linear operators and applications to partial differential equations*, **volume 44** Springer Science & Business Media, 2012.

[3] Weiming Wang, Yongli Cai, Mingjiang Wu, Kaifa Wang, and Zhenqing Li.r. . *Complex dynamics of a reaction–diffusion epidemic model. Nonlinear Analysis: Real World Applications*,, 13(5):2240–2258, 2012.

Dynamic Resource Optimization in 5G Networks: Integrating Mobile Edge Computing and Network Slicing

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Abstract: In modern smart factory systems, the customization of services through advanced technologies like Mobile Edge Computing (MEC) and Network Slicing is critical for managing diverse applications. These technologies enable efficient segmentation and allocation of heterogeneous resources on shared physical infrastructures, with a focus on supporting low-energy services. This paper presents the Factory Resource Optimization (FRO) framework, a novel approach that integrates MEC, Network Slicing, and a modified Non-Orthogonal Multiple Access (NOMA) strategy called Layered Multiple Access (LMA) to optimize resource allocation in 5G networks for smart factories. Unlike existing methods that often prioritize either energy efficiency or latency optimization, FRO addresses both simultaneously by formulating a non-convex combinatorial optimization problem. The proposed solution employs an iterative algorithm based on a decomposition methodology, offering a significant departure from traditional optimization techniques. This approach ensures efficient resource allocation while minimizing energy consumption for Equipment Monitoring Units (EMUs) and guaranteeing reliable, low-latency communication for Factory Safety Detectors (FSDs). Simulation results demonstrate significant energy savings for EMUs and compliance with FSD latency requirements, underscoring FRO's potential to enhance smart factory operations compared to current methods.

Keywords: Mobile Edge Computing, 5G Networks, Resource Allocation, Network Slicing, Non-Orthogonal Multiple Access, Smart Factories.

References

[1] A. Amzil, T. Elansari, M. Hanini, and A. Zaaloul. Optimizing Resource Allocation in 5G Networks: An Integrated Approach With Mobile Edge Computing, and Network Slicing. *Proceedings of the 2024 International Conference on Ubiquitous Networking (UNet)*, IEEE, Marrakech, Morocco, April 2024.

Novel Insights Into Cassava Mosaic Disease Using Caputo Fractional Derivative: Modeling And Analysis

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Abstract: In this work, we present a novel fractional-order model for the transmission dynamics of Cassava Mosaic Disease (CMD), using the Caputo derivative to account for memory effects intrinsic to disease processes. This innovative approach extends existing methodologies by capturing the long-term memory property of the disease, offering new insights into CMD dynamics. We rigorously establish the existence, uniqueness, positivity, and boundedness of the model's solutions, ensuring their biological relevance. A significant contribution of this work is the derivation of the basic reproduction number R0 using the next-generation matrix, followed by a detailed analysis of the local stability of equilibrium points through the Routh-Hurwitz criterion. Furthermore, a sensitivity analysis is conducted to identify critical parameters that most influence CMD transmissibility, providing essential guidance for targeted intervention strategies. Another key contribution is the implementation of numerical simulations via a generalized predictor-corrector approach, which vividly illustrates the influence of fractional-order derivatives on disease spread. The graphical results not only validate the theoretical results but also underscore the profound potential of fractional calculus in understanding CMD transmission and offer new perspectives and control strategies.

Keywords: Cassava mosaic disease, Fractional order derivative, Local stability, Sensitivity analysis, Numerical analysis.
2020 Mathematics Subject Classification: 26A33, 34A08, 92D30, 65L12, 37N25.

References

[1] S.S.HUMPHRIES, U.W.KHAN, A.PONGSUMPUN, AND SANGSAWANG, Sensitivity analysis of Cassava Mosaic Disease with saturation incidence rate model, AIMS Mathematics,8(2023),pp.6233–6254.

[2] N.SANTRA , D.SAHOO, S.MONDAL , AND G.SAMANTA, An epidemiological multi-delay model on Cassava Mosaic Disease with delay dependent parameters, Filomat,37(2023),pp.2887–2921.

[3] I .PODLUBNY, Fractional differential equations: an introduction to fractional derivatives,fractional differential equations, methods of their solution, and of their applications, vol.198,elsevier,1998.F.

Modélisation en épidémiologie

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Abstract: We begins with the well posedness and the positivity of the solution for the proposed model ,then the existence of equilibruim points and its L.A.S so as to developthe the G.A.S

$$\begin{cases} \frac{\mathrm{d}S(\mathrm{t})}{\mathrm{dt}} = A - (\nu_0+\omega)S(\mathrm{t}) - \mathcal{G}(S(\mathrm{t}),I(\mathrm{t})) + \int_0^{+\infty}\phi(a)\upsilon(\mathrm{t},a)\mathrm{d}a, \quad \mathrm{t}>0, \\ \frac{\partial\upsilon(\mathrm{t},a)}{\partial\mathrm{t}} + \frac{\partial\upsilon(\mathrm{t},a)}{\partial a} = -(\nu_0+\phi(a))\upsilon(\mathrm{t},a) - I(\mathrm{t})\beta(a)\upsilon(\mathrm{t},a), \quad t>0, \quad a>0, \\ \upsilon(\mathrm{t},0) = \omega S(\mathrm{t}), \quad \mathrm{t}>0, \\ \frac{\mathrm{d}I(\mathrm{t})}{\mathrm{dt}} = \mathcal{G}(S(\mathrm{t}),I(\mathrm{t})) + I(\mathrm{t})\int_0^{\infty}\beta(a)\upsilon(\mathrm{t},a)da + \gamma_1 R(t) - T(I(t)) - (\nu_0+\nu_1+\gamma_3)I(\mathrm{t}), \quad \mathrm{t}>0, \\ \frac{\mathrm{d}H(\mathrm{t})}{\mathrm{dt}} = T(I(t)) - (\nu_0+\nu_2+\gamma_2)H(\mathrm{t}), \quad \mathrm{t}>0, \\ \frac{\mathrm{d}R(\mathrm{t})}{\mathrm{dt}} = \gamma_3 I(\mathrm{t}) + \gamma_2 H(\mathrm{t}) - (\nu_0+\gamma_1)R(\mathrm{t}), \quad \mathrm{t}>0, \end{cases}$$

Keywords:
SIVR epidemic model , Vaccination age , Treatment , generalized nonlinear incidence rate , persistence , Lyapunov functional , global stability

References

[1] Van den Driessche. P. Watmough, *Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission,* ***180*** *(2002), 29–4.*

[2] H.L. Smith, H.R. Thieme, Dynamical Systems and Population Persistence*, AMS, Providence, 2011.*

[3] M. Iannelli, Mathematical Theory of Age-Structured Population Dynamics*, Applied Mathematics Monographs, vol. 7, Giardini Editorie Stampatori, Pisa, 1995.*

[4] Y. Chen, S. Zou and J. Yang, Global analysis of an SIR epidemic model with infection age and saturated incidence, Nonlinear Anal. Real World Appl. 30*, (2016), pp. 16-31.*

[5] J. Yang, M. Martcheva and L. Wang, Global threshold dynamics of an SIVS model with waning vaccine-induced immunity and nonlinear incidence, Math. Biosci. 268*, (2015), pp. 1-8.*

[6] K. Yasida, Functional Analysis*, 2nd edition,Springer-Verlag, Berlin-Heidelberg, New York, 1988.*

[7] W.M. Hirsch, H. Hanisch, and J.P. Gabriel, Differential equation models of some parasitic infections: Methods for the study of asymptotic behavior, Comm. Pure Appl. Math*, (1985), 733—753.*

[8] A. Ouakka, A. El Azzouzi and Z. Hammouch, Global dynamic behavior of a vaccination age-SVIR model with treatment and general nonlinear incidence rate, Journal of Computational and Applied Mathematics, 422*, (2023), 114848*

Some properties of generalized frames

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Abstract: This talk aims to study g-frames in Hilbert C^*-modules and investigate conditions under which the sum of two g-frames (or a g-frame and a g-Bessel sequence) remains a g-frame. We also address the stability of g-frames under certain perturbations and provide illustrative examples in the context of C^*-algebras. Our results unify and extend many of the existing theorems on g-frames, focusing on the invertibility of associated operators as a key condition for guaranteeing that sums of g-frames preserve the g-frame property.

Keywords: Frame, $g-$frame, $g-$Bessel sequence, Hilbert C^*-modules.
2020 Mathematics Subject Classification: 42C15; 46C05; 47B90.

References

[1] A. Lfounoune, R. El Jazzar, K-Frames in Super Hilbert C^*-Modules, Int. J. Anal. Appl., 23 (2025), 19. https://doi.org/10.28924/2291-8639-23-2025-19

[2] A. Lfounoune, H. Massit, A. Karara, M. Rossafi, Sum of g-Frames in Hilbert C^*-Modules, Int. J. Anal. Appl., 23 (2025), 64.

[3] M. Rossafi, F. D. Nhari, A. Touri, Continuous generalized atomic subspaces for operators in Hilbert spaces. J Anal (2024). https://doi.org/10.1007/s41478-024-00869-3

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Some properties of woven b-frames

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Abstract: This talk aims to study woven b-frames, which are a generalization of woven frames in Hilbert spaces, where the frames are generated by a bilinear mapping. We will define woven and weaving b-frames, present initial results, and explore their stability and preservation properties.

Keywords: Hilbert space, Banach space, b-frames, K-b-frames, woven b-frames, Woven K-b-frames.
2020 Mathematics Subject Classification: 46B15, 46A35, 42C15, 47B02, 47A07.

References

[1] M. Ismailov, F. Guliyeva, Y. Nasibov, On a Generalization of the Hilbert Frame Generated by the Bilinear Mapping J. Funct. Spaces, (2016).

[2] A. Karara, M. Rossafi, A. Touri, K-biframes in Hilbert spaces. J Anal 33, 235–251 (2025). https://doi.org/10.1007/s41478-024-00831-3

[3] M. Rossafi, H. Massit and C. Park, Weaving continuous generalized frames for operators, Montes Taurus J. Pure Appl. Math. 6 (1), 64-73, 2024; Article ID: MTJPAM-D-24-00026.

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Bridging Mathematical Epidemiology and Chemical Reaction Network Theory: A Novel Perspective

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Abstract: In this presentation, I will explore key concepts from Chemical Reaction Network (CRN) theory and Mathematical Epidemiology (ME), highlighting their intersection and mutual insights. This investigation leads us to propose, for the first time, a formal definition of ME models, which I will introduce. Additionally, I aim to raise awareness—particularly among researchers outside the ME community—of the utility of the next-generation matrix (NGM) approach for analyzing the stability of boundary equilibria, an aspect that remains underappreciated. Finally, I will showcase our Mathematica package, which facilitates these analyses.

Keywords: mathematical epidemiology; biochemical interaction network; essentially non-negative ODE systems; symbolic computation; Routh-Hurwitz stability conditions
2020 Mathematics Subject Classification: 34A34; 92B05; 34D20; 68V35; 92E20.

Investigation of the Performance of a Flow Energy Harvester Incorporating Time Delay Effects

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Abstract: This paper investigates the periodic energy harvesting (EH) of flow-induced vibrations in a nonlinear oscillator subjected to galloping instability under the influence of a time-periodic delay. The oscillator is coupled to an electrical circuit through a piezoelectric transduction mechanism, enabling the conversion of mechanical energy into electrical power. A key aspect of the study is the modulation of the time-delay amplitude, where the modulation frequency is tuned to be approximately twice the natural frequency of the systems oscillations.To analyze the system's response, the method of multiple scales is employed to derive an approximate analytical solution, providing an estimation of the amplitude of the periodic oscillations and the corresponding harvested power. The analytical results indicate that the presence of amplitude-modulated time delay in the mechanical system significantly enhances vibration amplitude, leading to a substantial increase in power output over a specific range of wind speeds. These theoretical findings are validated through numerical simulations, which confirm the efficiency of the proposed delay-based energy harvesting strategy.

Keywords: Energy harvesting, delayed oscillator, galloping excitation, piezoelectric coupling.

References

[1] Z. Ghouli, M. Hamdi, M. Belhaq, Energy harvesting from quasi-periodic vibrations using electromagnetic coupling with delay. Nonlinear Dyn, **89**, 1625-1636, (2017).

[2] Z. Ghouli, M. Hamdi, F. Lakrad, M. Belhaq, Quasiperiodic energy harvesting in a forced and delayed Duffing harvester device. Journal of Sound and Vibraton, **407**, 271-285, (2017).

[3] M. Belhaq, Z. Ghouli, M. Hamdi, Energy harvesting in a Mathieu-van der Pol-Duffing MEMS device using time delay. Nonlinear Dyn, **94**, 2537-2546, (2018).

[4] Z. Ghouli, M. Hamdi, M. Belhaq, The Delayed van der Pol Oscillator and Energy Harvesting. M. Belhaq (eds) Topics in Nonlinear Mechanics and Physics. Springer Proceedings in Physics, **228**, Springer, Singapore, (2019).

[5] Z. Ghouli, M. Hamdi, M. Belhaq, Energy Harvesting in a Duffing Oscillator with Modulated Delay Amplitude. I. Kovacic, S. Lenci (eds) IUTAM Symposium on Exploiting Nonlinear Dynamics for Engineering Systems. ENOLIDES 2018. IUTAM Bookseries, **37**, Springer, Cham, (2020).

[6] Z. Ghouli and M. Belhaq, Energy harvesting in a delay-induced parametric van der Pol-Duffing oscillator. Eur. Phys. J. Spec. Top., **230**, 3591-3598, (2021).

Optimal Strategies for Viable Production Systems: A Hamilton-Jacobi-Bellman Approach

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Abstract: This study addresses the maintenance of sustainable and efficient production systems in the face of logistical complexities, where pricing, investments in product quality, and advertising play a pivotal role in shaping market dynamics. We integrate control theory and Hamilton-Jacobi-Bellman (HJB) theory to bridge the geometric aspects with PDE analytical approaches. We aim to identify viable controls that help maintain sustainable and efficient production systems. A numerical study of sustainability scenarios demonstrates the practical utility of this approach, providing actionable insights for optimizing pricing, resource allocation, and production performance. The results highlight the framework's effectiveness in improving both sustainability and operational viability, offering a scalable and adaptable solution for managing dynamic production systems under real-world constraints.

Keywords: Hamilton-Jacobi-Bellman Equation; Optimal Control; Dynamic Systems; Decision-Making

References

[1] [Anton, R., Chenavaz, R. Y., & Paraschiv, C. (2023). Dynamic pricing, reference price, and price-quality relationship. Journal of Economic Dynamics and Control, 146, 104586.

[2] Bayen, T., & Rapaport, A. (2019). Minimal time crisis versus minimum time to reach a viability kernel: A case study in the prey-predator model. Optimal Control Applications and Methods, 40(2), 330-350.

[3] Chenavaz, R., & Paraschiv, C. (2018). Dynamic pricing for inventories with reference price effects. Economics, 12(1), 20180064.

[4] Aubin, J. P., Bayen, A. M., & Saint-Pierre, P. (2011). Viability theory: new directions. Springer Science & Business Media.

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Stochastic Optimal Control Problems with Jump Diffusion and Application

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Abstract: Near-optimization is as significant as optimization in stochastic control theory. We examine the near-optimal control of systems governed by stochastic differential equations with jumps (SDEJ), where the state and control variables influence the drift, diffusion, and jump components within a non-convex admissible set. Using the $\mathcal{H}$-function, Ekeland's variational principle, stability properties of the state, and the first and second adjoint processes, we derive necessary and sufficient conditions for near-optimality. Additionally, we extend our results to stochastic singular control problems.

As an application, we propose two control strategies to mitigate the spread of COVID-19 within an epidemic model and present optimal strategies explicitly for a quadratic case. Finally, numerical simulations are provided to support our theoretical findings, illustrating the effectiveness of an optimized combination of vaccination and treatment strategies to understand the challenges posed by COVID-19 in Brazil.

Keywords:
2020 Mathematics Subject Classification:

References

The Extinction and Persistence in mean for stochastic SIRH epidemic model with relapse, healing and vaccination using Lévy noise perturbation.

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Abstract: In this work, we propose and analyze a stochastic SIRH (Susceptible-Infected-Recovered-Healed) epidemic model with healing, relapse, and jump effects to investigate the impact of vaccination on the disease dynamics. The stochastic perturbations are introduced into the model to account for the inherent randomness in the disease transmission and recovery processes. The existence and uniqueness of the positive global solution for the stochastic model are established. Further, we study the conditions for the extinction and persistence of the disease.

The impact of vaccination is incorporated into the model by considering the vaccination rate as a control parameter. Numerical simulations are carried out to illustrate the dynamical behavior of the stochastic SIRH model and to analyze the effects of vaccination on the disease spread.

Keywords: Stochastic epidemic model with relapse, healing and jumps, Vaccination, Extinction, Persistence in mean.

2024 Mathematics Subject Classification.

References

[1] Diekmann, Odo and Heesterbeek, Johan Andre Peter and Metz, Johan AJ. On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations. *Journal of mathematical biology. Springer*, **Vol** (28): 365–382, 1990.

[2] Dietz, Klaus and Heesterbeek, JAP, Daniel Bernoulli's epidemiological model revisited,*Mathematical biosciences. Elsevier.*, **Vol** 180: 1–21, 2002.

[3] Hethcote, Herbert W, The mathematics of infectious diseases,*SIAM review.*, **Vol** 4: 599–653, 2000.

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TPFA finite volume scheme for the two-dimensional viscous Burgers' equation

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Abstract: This paper presents a methodological approach based on the Two-Point Flux Approximation (TPFA) finite volume method [1] for solving the two-dimensional viscous Burgers' equation (1)[2]. A semi-implicit discretization is applied to the nonlinear viscous Burgers' equation. The approach leverages the simplicity of the TPFA method, while avoiding the need to solve the nonlinear system arising from the nonlinear term in the equation. The accuracy and efficiency of the proposed numerical scheme are demonstrated through several tests, where the chosen time step depends only on the square of the spatial step size. Super-convergence is observed for different values of Reynolds numbers in both the discrete L^2 and L^∞-norms.

$$\begin{cases} \partial_t u + u\partial_x u + \mathrm{v}\partial_x u - \frac{1}{Re}\mathrm{div}(\nabla u) = f_1 & on \quad (0,T)\times\Omega \\ \partial_t v + u\partial_x v + \mathrm{v}\partial_x v - \frac{1}{Re}\mathrm{div}(\nabla v) = f_2 & on \quad (0,T)\times\Omega \\ \mathrm{u}(x,0) = \mathrm{u}_0(x),\ \mathrm{v}(x,0) = \mathrm{v}_0(x) & on \quad (0,T)\times\partial\Omega \\ \mathrm{u}(x,0) = \mathrm{u}_0(x),\ \mathrm{v}(x,0) = \mathrm{v}_0(x) & on \quad \Omega \end{cases} \tag{1}$$

where Re is the Reynolds number, Ω is the spatial domain, and T is the final time.

Keywords: TPFA, Semi-implicit, Super-convergence, Burgers' equation
2020 Mathematics Subject Classification: 65N08, 65N22.

References

[1] Eymard, R., Gallouët, T., Herbin, R. Finite volume methods. *Handbook of numerical analysis*, **7**, 713-1018, 2000.

[2] Wang, Y., Navon, I. M., Wang, X., Cheng, Y., 2D Burgers equation with large Reynolds number using POD/DEIM and calibration. *International Journal for Numerical Methods in Fluids*, **82(12)**, 909-931, 2016.

A mathematical study of combined treatments for breast cancer

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Abstract: In this study, we present a mathematical model for breast cancer treatment that combines chemotherapy and a ketogenic diet. We analyze the treatment dynamics using control theory and apply Pontryagin's maximum principle to characterize the optimal treatment strategies. Numerical simulations are conducted to illustrate and support the theoretical findings.

Keywords: Breast cancer, chemotherapy, ketogenic diet, Pontryagin's maximum principle, optimal control.
2020 Mathematics Subject Classification:

References

[1] WHO. Morocco Records Over 50,000 New Cases of Cancer Every Year. Editor: Morocco, 2024.

[2] H. AKIL, N. YOUSFI. A mathematical model of breast cancer (er+) with excess estrogen: A mixed treatment using ketogenic diet, endocrine therapy, and immunotherapy. *Commun. Math. Biol.Neurosci.*, 2023.

[3] K. HATTAF, N. YOUSFI. Optimal Control of a Delayed HIV Infection Model with Immune Response Using an Efficient Numerical Method. International Scholarly Research Network.. *International Scholarly Research Network.*, 2012.

On controlled K-g-Fusion Frames within Hilbert C^*-Modules

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Abstract: In this talk, we present several aspects of controlled K-g-fusion frames in Hilbert C^*-modules. We provide detailed characterizations of these frames, highlighting their structural properties. Furthermore, we explore some properties of controlled K-g-fusion frames.

Keywords: Hilbert, C^*-Modules, K-g-Fusion Frames, Controlled K-g-Fusion Frames, Frames,g-Fusion Frames.
2020 Mathematics Subject Classification: Primary 42C15; Secondary 46L05.

References

[1] V.M. Manuilov, E.V. Troitsky. *Hilbert C^*-modules*. Vol. 226, American Mathematical Soc., 2005.

[2] F. D. Nhari, R. Echarghaoui, M. Rossafi. K-g-fusion frames in Hilbert C*-modules. *Int. J. Anal. Appl*, **Vol 19** : 836–857, 2021.

[3] M. Rossafi, F. D. Nhari. Controlled K- g- Fusion Frames in Hilbert C^*- Modules. *International Journal of Analysis and Applications*, **Vol 20** : 1–20, 2022.

Local integrability and Riesz measure of $G(\cdot)$-superharmonic functions

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Abstract: In this talk, we study the local integrability properties of superharmonic functions associated with partial differential equations under Musielak-Orlicz growth conditions in Lebesgue and Musielak-Orlicz spaces. We investigate the relationship between the Riesz measure μ and $G(\cdot)$-superharmonic functions u, which satisfy the equation

$$-\operatorname{div} \mathcal{A}(x, Du) = \mu$$

in the distributional sense, where $\mathcal{A}(x,\xi)\cdot\xi \approx G(x,|\xi|)$, and $G(\cdot)$ is a Φ-function.

Keywords: $G(\cdot)$-superharmonic, Local integrability, Musielak-Orlicz growth, Φ-function, Riesz measure.
2020 Mathematics Subject Classification: 35B65, 35J62, 46E30.

References

[1] Harjulehto, P., Hästö, P.: Boundary regularity under generalized growth conditions. Zeitschrift für Analysis und ihre Anwendungen. **38**, 73–96 (2019).

[2] Harjulehto, P., Hästö, P.: Orlicz Spaces and Generalized Orlicz Spaces. Springer-Verlag, Cham (2019).

[3] Heinonen, J., Martio, O., Kilpeläinen, T.: Nonlinear Potential Theory of Degenerate Elliptic Equations. Clarendon Press, New York (1993).

[4] Lukkari, T.: Elliptic equations with nonstandard growth involving measures. Hiroshima Math. J. **38**(1), 155-176 (2008).

[5] Benyaiche, A., Khlifi, I.: PWB-method and Wiener criterion for boundary regularity under generalized Orlicz growth. Positivity **27**, 24 (2023). https://doi.org/10.1007/s11117-023-00979-5.

[6] Kilpeläinen, T., Malý, J.: Degenerate elliptic equations with measure data and nonlinear potentials. Annali della Scuola Normale Superiore di Pisa, Classe di Scienze. **19**, 591-613 (1992).

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Log-volatility Models for Modelling Financial Return Series in the Presence of Zeros: a Comparative Study

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Abstract: In this work, we address the challenges posed by zero returns in both log-GARCH and stochastic volatility (SV) models, with a particular focus on their asymmetric variants. Building upon previous imputation techniques for handling zero returns, as discussed in [1,2, 3], we propose a unified approach that enhances parameter estimation accuracy for both model classes. Specifically, we employ the Quasi-Maximum Likelihood (QML) estimation method, incorporating the Kalman filter for both the asymmetric log- GARCH and asymmetric SV models, to ensure robust parameter estimation even in the presence of zero returns. A comparative study is conducted on a set of financial return series containing zeros, examining the performance of the models in question using our proposed estimation method.

Keywords: log-GARCH, SV, Kalman filter
2020 Mathematics Subject Classification: Primary 62M10, 62P05, 91G70.

References

[1] Francq C., Wintenberger O., Zakoian J.M. (2013) GARCH models without positivity constraints: Exponential or log GARCH?. J. Econom. 177(1): 34-46

[2] Sucarrat G., Escribano A. (2013) Unbiased QML estimation of log-GARCH models in the presence of zero returns.

[3] Settar A., Badaoui M. (2021) On the treatment of zero returns in the estimation of log-GARCH model: empirical study. IOP Conf. Ser.: Mater. Sci. Eng. IOP Publishing, p. 012029.

RBDSDEs Driven by Teugels Martingales Associated to a Lévy Process with Discontinuous Barrier

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Abstract: In this talk, we present a generalization of Reflected Backward Doubly Stochastic Differential Equations (RBDSDEs) driven by Teugels martingales associated to a Lévy process. In our setting, the associated barrier is no longer càdlàg, but only assumed to be optional. We prove existence and uniqueness of the solution using a fixed point argument

Keywords: Reflected backward doubly stochastic differential equations; Teugels mar- tingales; Mertens decomposition; Strong optional supermartingale
2020 Mathematics Subject Classification: 60H10; 60G40; 60H05; 60G51

References

[1] B. Berrhazi, M. El Fatini, A. Hilbert, N. Mrhardy, and R. Pettersson. RBDSDEs driven by Teugels Martingales with Discontinuous Barrier. *Markov Processes Relat. Fields*, **29** (3): 329–346, 2023.

[2] B. Berrhazi, M. El Fatini, A. Hilbert, N. Mrhardy, and R. Pettersson. RBDSDEs with jumps and optional Barrier and mean Field game with common noise. *Stochastics*, **95** (4): 615–634, 2023.

[3] B. Berrhazi, M. El Fatini, A. Hilbert, N. Mrhardy, and R. Pettersson. Reflected backward doubly stochastic differential equations with discontinuous barrier. *Stochastics*, **92** (7): 1100–1124, 2020.

[4] M. Grigorova, P. Imkeller, E. Offen, Y. Ouknine, and M.C. Quenez. Reflected BSDEs when the obstacle is not right-continuous and optimal stopping. *The Annals of Applied Probability*, **27** (5): 3153–3188, 2017.

A Mathematical Model for the Impacts of Vaccination and Quarantine on the Dynamics of COVID-19 Pandemic: Deterministic and Stochastic Analysis

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Abstract: The use of mathematical analysis in understanding epidemics is essential in predicting the progression of diseases over time and in providing direction to decision-makers with respect to public health policy. It is within this context that the purpose of this work is to study a stochastic model of COVID-19 disease dynamics. In our SIR mathematical model, the transmission of the infection from infected individuals to susceptible individuals occurs at a rate , which is disrupted by white noise. Our work starts with the investigation of the well-posedness of the mathematical model. Then, we present the sufficient conditions for the COVID-19 extinction and persistence in mean of this pandemic. It moves then to the investigation of certain numerical findings to support the theoretical analysis and to show the effectiveness of quarantine strategy in controlling the COVID-19 pandemic. The theoretical findings and the numerical results align well. Finally, we show numerically the effectiveness of quarantine strategy in controlling the spread of COVID-19 infection. Controlling the infection severity may help to achieve some of sustainable development goals.

Keywords: Global stability, Extinction, Persistence in mean, Covid-19 infection.
2020 Mathematics Subject Classification: 37H05, 37H10, 37H30, 60H10

References

[1] W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, **115** (772): 700–721, 1927.

[2] M. Sadki, A. Ez-zetouni, and K. Allali. Persistence and extinction for stochastic HBV epidemic model with treatment cure rate. *Boletim da Sociedade Paranaense de Matemática*, **42**, Article in press, 2024. doi:10.5269/bspm.64254.

[3] M. Sadki, Z. Yaagoub, and K. Allali. Stochastic analysis of two-strain epidemic model with non-monotone incidence rates: Application to COVID-19 pandemic. *Mathematical Methods in the Applied Sciences*, **47** (11): 8651–8668, 2024.

Extinction, ergodicity and stationary distribution of an SEI epidemic model with stochastic perturbation

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Abstract: In this study, we analyze a stochastic SEIS epidemic model with nonlinear innate immunity under the influence of white noise. We first prove the existence and uniqueness of a global positive solution for the proposed system, ensuring the well-defined behavior of the model over time. Then, we established sufficient conditions for the extinction of the infectious disease. Moreover, by formulating a suitable stochastic Lyapunov function, we establish sufficient conditions for the existence and uniqueness of an ergodic stationary distribution of the model's solution. Finally, the theoretical results are verified by some numerical simulations.

Keywords: Stochastic epidemic model, Extinction, Persistence, ergodicity, Stationary distribution.
2020 Mathematics Subject Classification: 34A37, 34D05, 92D30 .

References

[1] S. Jain, S. Kumar. Dynamic analysis of the role of innate immunity in SEIS epidemic model. *Eur. Phys J. Plus*, **136** (439): 1–12, 2021.

[2] J. M. Liu and F. Y. Wei. Dynamics of stochastic SEIS epidemic model with varying population size. *Physica A: Statistical Mechanics and Its Applications*, **464** (241-250): 241–250, 2016.

[3] M. Naim, F. Lahmidi, and A. Namir. Threshold dynamics of an SEIS epidemic model with nonlinear incidence rates. *Differential Equations and Dynamical Systems*, pp. 1–14, 2021.

[4] M. Y. Song, W. J. Zuo, D. Q. Jiang, and T. Hayat. Stationary distribution and ergodicity of a stochastic cholera model with multiple pathways of transmission. *Journal of the Franklin Institute*, **357** (15): 10773–10798, 2020.

[5] R. Khasminskii. *Stochastic Stability of Differential Equations*. Editor: Springer-Verlag, Berlin Heidelberg, Germany, 2nd edition, 2012.

Modeling and investigating malaria P. Falciparum and P. Vivax infections: Application to Djibouti data

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Abstract: Malaria is an infectious and communicable disease, caused by one or more species of Plasmodium parasites. There are five species of parasites responsible for malaria in humans, of which two, Plasmodium Falciparum and Plasmodium Vivax, are the most dangerous. In Djibouti, the two species of Plasmodium are present in different proportions in the infected population: 77% of P. Falciparum and 33% of P. Vivax. In this study we present a new mathematical model describing the temporal dynamics of Plasmodium Falciparum and Plasmodium Vivax co-infection. We focus briefly on the well posedness of this model and on the calculation of the basic reproductive numbers for the infections with each Plasmodium species that help us understand the long-term dynamics of this model (i.e., existence and stability of various eqiuilibria). Then we use computational approaches to: (a) identify model parameters using real data on malaria infections in Djibouti; (b) illustrate the influence of different estimated parameters on the basic reproduction numbers; (c) perform global sensitivity and uncertainty analysis for the impact of various model parameters on the transient dynamics of infectious mosquitoes and infected humans, for infections with each of the Plasmodium species. The originality of this research stems from employing the FAST method and the LHS method to identify the key factors influencing the progression of the disease within the population of Djibouti. In addition, sensitivity analysis identified the most influential parameter for Falciparium and Vivax reproduction rates. Finally, the uncertainty analysis enabled us to understand the variability of certain parameters on the infected compartments.

Keywords: Falciparum and vivax models, Reproduction number, Stability analysis, Sensitivity analysis, FAST method, LHS method, Djibouti data.
2020 Mathematics Subject Classification: 92D30, 37N25, 34D20, 92C60.

References

[1] Souleiman, Y. , Mohamed, A. & Ismail, L. Analysis the Dynamics of SIHR Model: Covid-19 Case in Djibouti. *Applied Mathematics*, **12**, (10), 867-881, (2021).

[2] Massard, M., Eftimie, R., Perasso, A., & Saussereau, B. A multi-strain epidemic model for COVID-19 with infected and asymptomatic cases: Application to French data. *Journal of Theoretical Biology*, **545**, p.111117, 2022.

[3] Mahato, B., Mishra, B. K., Jayswal, A., & Mishra, B. K. Mathematical model of malaria for Co-infection of plasmodium vivax and plasmodium falciparum in India. Microbiology Research Journal, **vol. 5**, (3), p. 285, 2015.

An abstract semilinear functional integrodifferential equations.] An abstract partial functional integrodifferential equations
Koumla]

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Abstract

In this work, we investigate the existence and regularity of solutions for some partial functional integrodifferential equations with finite delay. The continuous dependence upen initial values and asymptotic stability are also studied. Firstly, we show the existence of the mild solutions. Secondly, we give sufficient conditions ensuring the existence of the strict solutions. The method used treats the equations in the domain of A with the graph norm employing results from linear semigroup theory. To illustrate our abstract result, we conclude this work with an application.

keywords and phrases: Mild and strict solutions, partial functional integrodifferential equations, C_0-semigroup, infinitesimal generator, finite delay, phase space.

Wold-type Decomposition for doubly commuting $n-$tuples.

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Abstract: In this work, we study the class of doubly commuting $n-$tuples satisfying the wandering subspace property, the Beurling-type theorem or admitting a Wold-type decomposition. It is shown that a doubly commuting $n-$tuple $\mathbf{T} = (T_1, \cdots, T_n)$ satisfies the wandering subspace property if and only if T_i does for every $1 \leq i \leq n$. Finally, in the case where T_i admits a Wold-type decomposition for every i, we exhibit a Wold-type decomposition for the doubly commuting tuples $(T_1, \cdots, T_n)$.

Keywords: Doubly commuting tuples, Wandering subspace property, Beurling-type theorem, Wold-type decomposition, Doubly commuting invariant subspace.
2020 Mathematics Subject Classification: Primary: 47A13. Secondary 47A15, 47B37.

References

[1] A. Aleman, S. Richter, C. Sundberg. Beurling's Theorem for the Bergman space. Acta Math, **177.2**(1996), 275-310.

[2] Z. Burdak, M. Kosiek, M. Słociński. The canonical Wold decomposition of commuting isometries with finite dimensional wandering spaces. Bulletin des Sciences Mathématiques, **137.5** (2013), 653-658.

[3] H. Ezzahraoui, M. Mbekhta, E.H. Zerouali. Operator inequalities related to Beurling type theorem. Int. J. Funct. Anal. Oper. Theory and Appl, **7** (2015), 107-119.

[4] S. Richter. Invariant subspaces of the Dirichlet shift. J. Reine Angew. Math, **386** (1988), 205–220.

[5] J. Sarkar. Wold decomposition for doubly commuting isometries. Linear Algebra and its Applications, **445** (2014), 289-301.

[6] S. Shimorin. Wold-type decompositions and wandering subspaces for operators close to isometries. J. Reine Angew. Math, **531** (2001), 147-189.

Positive solutions to multi-critical elliptic problems

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Abstract: Abstract In this paper, we investigate the existence of multiple positive solutions to the following multi-critical elliptic problem

$$\begin{cases} -\Delta u = \lambda |u|^{p-2}u + \sum_{i=1}^{k} \left(|x|^{-(N-\alpha_i)} * |u|^{2_i^*} \right) |u|^{2_i^*-2}u \text{ in } \Omega, \\ u \in H_0^1(\Omega) \end{cases} \tag{1}$$

in connection with the topology of the bounded domain $\Omega \subset \mathbb{R}^N, N \geq 4$, where $\lambda > 0$, $2_i^* = \frac{N+\alpha_i}{N-2}$ with $N-4 < \alpha_i < N, i = 1, 2, \cdots, k$ are critical Hardy-Littlewood-Sobolev exponents and $2 < p < 2^* = \frac{2N}{N-2}$. We show that there is $\lambda^* > 0$ such that if $0 < \lambda < \lambda^*$ problem (1) possesses at least $\text{cat}_\Omega(\Omega)$ positive solutions. We also study the existence and uniqueness of positive solutions for the limit problem of (1).

Keywords: Multi-critical problem, Multiple solutions, Elliptic equation.
2020 Mathematics Subject Classification: Primary 35B33, 35B38, 35J20, 35J60.

References

[1] Alves, C.O., Ding, Y.H.: Multiplicity of positive solutions to a p-Laplacian equation involving critical nonlinearity. J. Math. Anal. Appl. 279, 508–521 (2003)

[2] Bahri, A., Coron, J.M.: On a nonlinear elliptic equation involving the Sobolev exponent: the effect of the topology of the domain. Commun. Pure Appl. Math. 41, 253–294 (1988)

[3] Benci, V., Cerami, G.: Positive solutions of some nonlinear elliptic problems in exterior domains. Arch. Ration. Mech. Anal. 99, 283–300 (1987)

Infinitely many solutions for an elliptic equation in divergent form with critical Sobolev exponent and concave-convex nonlinearity

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Abstract: By using concentration estimates, Fountain Theorem and its Dual form we prove the existence of two disjoint and infinite sets of solutions for an elliptic equation in divergent form with critical growth and concave-convex nonlinearities. The problem is considered in a bounded domain under appropriate conditions on N and the coefficient of the critical nonlinearity.

Keywords: Infinitely many solutions,semilinear elliptic equations, Concave-convex, Fountain Theorem, Dual fountain Theorem.
2020 Mathematics Subject Classification: Primary 35J20, 35J70.

References

[1] Ambrosetti, A., Brezis, H., Cerami, G.: Combined effects of concave and convex nonlinearities in some elliptic problems. J. Funct. Anal. 122, 519–543 (1994)

[2] Ambrosetti A, Struwe M. A note on the problem $-\Delta u = \lambda u + u|u|^{2^*-2}$. Manuscripta Math, 54: 373-379 (1986)

[3] Barrios, B., Colorado, E., Servadei, R., Soria, F.: A critical fractional equation with concave-convex power nonlinearities. Ann. Inst. Henri Poincaré, Anal. Non Linéaire 32, 875–900 (2015)

A Study of Multiple Solutions for a Bi-Nonlocal Fractional Kirchhoff Equation Involving Variable Order

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Abstract: We investigate the multiplicity of solutions for the bi-nonlocal fractional Kirchhoff problem with variable order and variable exponents via Krasnoselskii's genus:

$$\begin{cases} M\left(\int_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^{p(x,y)}}{p(x,y)|x-y|^{N+s(x,y)p(x,y)}}\mathrm{d}x\mathrm{d}y\right)(-\Delta)^{s(x)}_{p(x)}\, u = f(x,u)\left[\int_\Omega F(x,u)\right]^r & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N\backslash\Omega, \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ with $1 < p(x) < N$ and $N \geq 2$, M and f are continuous functions, f is an odd function, $F(x,u) = \int_0^u f(x,\xi)d\xi$, and $r > 0$ is a real parameter.

Keywords: Kirchhoff equation, concave-convex term, Sobolev space with variable exponent, variable orders, variational methods, Krasnoselskii's genus.
2020 Mathematics Subject Classification: 35B30, 35B38, 35B40, 35J92, 35J60, 35R11.

References

[1] A. Bahrouni, Comparison and sub-supersolution principles for the fractional $p(x)$–Laplacian, J. Math. Anal. Appl. 458, 1363–1372 (2018).

[2] B. Cekic, R.A. Mashiyev, Existence and localization results for $p(x)$–Laplacian via topological methods, Fixed Point Theory Appl. (2010).

[3] Corrêa, F.J., Figueiredo, G.M.: On an elliptic equation of p-Kirchhoff type via variational methods. Bull. Aust. Math. Soc. 74(2), 263-–277 (2006).

[4] F. J. S. A. Correa e A. C. dos Reis Costa, On a p(x)-Kirchhoff equation with critical exponent and an additional term via truncation argument, Math. Nachr. 288 (2015) 1226-1240.

[5] F.J.S.A. Corrêa, G.M. Figueiredo, On a p-Kirchhoff equation via Krasnoselskii's genus, Appl.Math.Lett.22 819–822 (2009).

[6] G. Kirchhoff, Mechanik, Teubner, Leipzig (1883).

[7] J. Zuo, L. Yang, S. Liang, A variable-order fractional $p(x)$-Kirchhoff type problem in $\mathbb{R}^N$, Mathematical Methods in the Applied Sciences, **44**, 3872–3889 (2021).

Multiple Solutions to the Fractional (p, q)-Laplacian Equations involving the critical exponents

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Abstract: In this article, we consider the following fractional (p, q)-Laplacian equation with critical exponent

$$(-\Delta_p)^{s_1} u + (-\Delta_q)^{s_2} u = \lambda g(x)|u|^{r-2}u + h(x)|u|^{p_{s_1}^*-2}u \text{ in } \mathbb{R}^N,$$

where $0 < s_2 < s_1 < 1$, $1 < q \leq p < r < p_{s_1}^*$ and $p_s^* := \frac{Np}{N-ps}$ for any $s \in (0, 1)$. Under certain assumptions on g et h, using an abstract critical point theorem from [3], we obtain a multiple solutions for λ sufficiently large. A similar problem with subcritical exponents is also considered.

Keywords: Fractional (p, q)-Laplacian, Variational methods, Concentration compactness principles, Palais-Smale condition, Critical point theorem.
2020 Mathematics Subject Classification: Primary 35J47, 35J50, 35J60, 35Q55, 35Q40.

References

[1] Mingione, G., Rădulescu, V.D.: Recent developments in problems with nonstandard growth and nonuniform ellipticity. J. Math. Anal. Appl. **501**, 125197 (2021)

[2] Papageorgiou, N.S., Rădulescu, V.D., Repovš, D.D.: Ground state and nodal solutions for a class of double phase problems. Z. Angew. Math. Phys. **71**, 15 (2020)

[3] Perera, K.: Abstract multiplicity theorems and applications to critical growth problems. J. Anal. Math.(in press) (2023)

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The effect of topology on the number of positive solutions for upper critical Choquard equation

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Abstract: In this article, we consider the following Choquard equation with upper critical exponent:

$$-\Delta u = \mu f(x)|u|^{p-2}u + g(x)\left(I_\alpha * \left(g|u|^{2^*_\alpha}\right)\right)|u|^{2^*_\alpha - 2}u, \quad x \in \Omega,$$

where $\mu > 0$ is a parameter, $N > 4, 0 < \alpha < N, I_\alpha$ is the Riesz potential, $\frac{N}{N-2} < p < 2, \Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, and f and g are continuous functions. For μ small enough, using variational methods, we establish the relationship between the number of solutions and the profile of potential g.

Keywords: Choquard equation, critical exponent, multiple solutions, variational methods.
2020 Mathematics Subject Classification: Primary 35A15, 35B40, 35J20.

References

[1] E. Lieb, Existence and uniqueness of the minimizing solution of Choquard's nonlinear equation, Studies Appl. Math. 57 (1976/77), no. 2, 93-105.

[2] S. Pekar, Untersuchung über die Elektronentheorie der Kristalle, Akademie Verlag, Berlin, 1954.

[3] R. Penrose, On gravity's role in quantum state reduction, Gen. Relativ. Gravitat, 28 (1996), 581-600.

Multiple positive solutions for a critical fractional p-Laplacian system with concave nonlinearities

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Abstract: The aim of this paper is to study the following nonlinear fractional p-Laplacian system with critical exponents:

$$\begin{cases} (-\Delta)_p^s u + |u|^{p-2}u = \lambda g(x)|u|^{q-2}u + \frac{\alpha}{\alpha+\beta} f(x)|u|^{\alpha-2}u|v|^{\beta} & \text{in } \Omega \\ (-\Delta)_p^s v + |v|^{p-2}v = \mu h(x)|v|^{q-2}v + \frac{\beta}{\alpha+\beta} f(x)|u|^{\alpha}|v|^{\beta-2}v & \text{in } \Omega \\ u = v = 0 & \text{in } \mathbb{R}^N \backslash \Omega \end{cases}$$

where Ω is a smooth bounded set in $\mathbb{R}^N, 0 < s < 1, \lambda, \mu > 0$ are two parameters, $1 < q < p < p_s^*, N > ps, \alpha, \beta > 1$ satisfy $\alpha + \beta = p_s^*$ with $p_s^* = \frac{np}{n-ps}$ is the fractional Sobolev critical exponent and $(-\Delta)_p^s$ is the fractional p-Laplacian operator. Using the Nehari manifold and Ljusternik-Schnirelmann category, we study the topology of the global maximum set Θ of $f(x)$, and show that the system has at least at least $\mathrm{cat}_{\Theta_\delta}(\Theta) + 1$ distinct positive solutions.

Keywords: Fractional p-Laplacian; critical elliptic sysytem; multiplicity; concave-convex nonlinearities; Lusternik–Schnirelmann category
2020 Mathematics Subject Classification: Primary 35J50; 35B33; 47G20.

References

[1] Benmouloud S, Echarghaoui R, Sbaï SM. Multiplicity of positive solutions for a critical quasilinear elliptic system with concave and convex nonlinearities. J Math Anal Appl. 2012;396:375–385.

[2] Echarghaoui R, Khouakhi M, MMasmodi M. Existence and multiplicity of positive solutions for a class of critical fractional Laplacian systems. J Elliptic Parabol Equ. 2022;8:813–835.

[3] Echarghaoui R, Masmodi M. Two disjoint and infinite sets of solutions for a concaveconvex critical fractional Laplacian equation. Fract Calc Appl Anal. 2022;25:1604–1629.

New multiplicity results for critical p-Laplacian problems

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Abstract: We prove new multiplicity results for the Brézis–Nirenberg problem for the pLaplacian. Our proofs are based on a new abstract critical point theorem involving the $\mathbb{Z}_2$-cohomological index that requires less compactness than the (PS) condition.

Keywords: Critical p-Laplacian problems, Multiplicity results, Abstract critical point theorems, $\mathbb{Z}_2$-cohomological index.
2020 Mathematics Subject Classification: Primary 35J92.

References

[1] H. Brézis and L. Nirenberg, Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents, Communications on Pure and Applied Mathematics, 36 (4) (1983) 437–477.

[2] F.V. Atkinson, H. Brézis, and L.A. Peletier, Solutions d'équations elliptiques avec exposant de Sobolev critique qui changent de signe, C. R. Acad. Sci., Sér. I Math., 306 (16) (1988) 711–714.

[3] K. Perera, Nontrivial critical groups in p-Laplacian problems via the Yang index, Topological Methods in Nonlinear Analysis, 21 (2) (2003) 309–318.

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A study of a delayed stochastic SIR model with general incidence and logistic growth

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Abstract: Starting by introducing a stochastic SIR model which has four different parameters. Logistic growth to make the model more realistic. Adding an non-linear saturated incidence rate for more understanding to the disease's burden, and adding also a constant to represent the significant part played by media coverage, as experienced through the past pandemic, in reducing the contact rate of infected people. Then, a term for representing disease latency. Firstly, we show the existence of a solution for this stochastic model, its uniqueness and its positivity, hence establishing the basis of solid theoretical framework. As a main part of this work, we prove the extinction of the disease under some conditions. Moreover, we show its persistence in mean. To approve our theoretical results, some numerical simulations are made for extinction and persistence scenarios. As a conclusion of this study, a discussion is given as numerical simulations to discuss the effect of media coverage and the saturation coefficient that measures the psychological impact.

Keywords: Delayed Stochastic model, Logistic growth, Media coverage, Extinction, Persistence in mean.
2020 Mathematics Subject Classification: Primary 39A50, 37A50, 34K50, 60H05, 60H20.

References

[1] Mao, X. *Stochastic differential equations and applications*. Elsiver, 2007.

[2] Guo, W., Cai, Y., Zhang, Q., Wang, W. Stochastic persistence and stationary distribution in an sis epidemic model with media coverage. *Physica A : Statistical Mechanics and its Applications*, **492** : 2220–2236, 2018.

Stochastic epidemic model with relapse

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Abstract: In this work, we consider a stochastic epidemic model with relapse. We prove the existence and the uniqueness of the positive solution. We establish sufficient conditions for the extinction and the persistence in mean of the stochastic system. Moreover, we also establish sufficient conditions for the existence of ergodic stationary distribution to the model, which reveals that the infectious disease will persist. The graphical illustrations of the approximate solutions of the stochastic epidemic model have been performed.

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Keywords:Stochastic SIR epidemic with Relapse; Extinction; Persistence; Markov Semigroup; Stationary distribution.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] B-E. Berrhazi, M. El Fatini, T. Caraballo, R. Pettersson. A stochastic SIRI epidemic model with Lévy noise. Discrete Contin. Dyn. Syst. Ser. B 23 (2018), no. 6, 2415-2431.

[2] L. Bavagnoli et al. antiviral Identification of a novel antiviral micro-RNA targeting the NS1 protein of the H1N1 pandemic human influenza virus and a corresponding viral escape mutation. Antiviral Research 171 (2019).

[3] B-E. Berrhazi, M. El Fatini, R. Pettersson, A. Laaribi. Media effects on the dynamics of a stochastic SIRI epidemic model with relapse and Lévy noise perturbation. Int. J. Biomath. 12 (2019), no. 3, 1950037, 21 pp.

Long time behaviour and quasi-density function for a stochastic epidemic model with relapse and reinfection

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Abstract: A stochastic SIRI (Susceptible-Infected-Recovered-Infected) epidemic model with relapse and reinfection is established in this paper. Firstly, we prove that the solution to the epidemic model is unique and global positive. Next, we determine some sufficient conditions for the extinction of the disease when $\mathcal{R}_0^s < 1$ and for the persistence in mean in the case of $\bar{\mathcal{R}}_0^s > 1$. Furthermore, we prove the existence of at least one ergodic stationary distribution of the stochastic model if $\bar{\mathcal{R}}_0^s > 1$. Additionally, by solving the corresponding three-dimensional Fokker-Planck equation, it is theoretically show that the epidemic model has a log-normal probability density function when $\bar{\mathcal{R}}_0^s > 1$, then we obtain the exact expression of density function of the stationary distribution. Finally, we give some numerical simulations to support our theoretical results.

Keywords: Stochastic SIRI epidemic model, Extinction, Persistence in mean, Ergodic stationary distribution, Fokker-Planck equation, Probability density function.
2020 Mathematics Subject Classification: Primary 92B05, 60G53, 37A00, 60G10, 60E05, 35Q84.

References

[1] B. Zhou, D. Jiang, Y. Dai, T. Hayat and A. Alsaedi. Stationary distribution and probability density function of a stochastic SVIS epidemic model with standard incidence and vaccination strategies. *Chaos Solitons Fractals*, **143**: 110601, 2021.

[2] M. El Fatini, M. El Khalifi, R. Gerlach, A. Laaribi and R. Taki. Stationary distribution and threshold dynamics of a stochastic sirs model with a general incidence. *Physica A: Statistical Mechanics and its Applications*, **534**: 120696, 2019.

[3] T. Caraballo, M. El Fatini, M. El Khalifi and A. Rathinasamy. Analysis of a stochastic coronavirus (COVID-19) Lévy jump model with protective measures. *Stochastic Analysis and Applications*, **41** (1): 45–59, 2023.

[4] B. Zhou, B. Han and D. Jiang. Ergodic property, extinction and density function of a stochastic SIR epidemic model with nonlinear incidence and general stochastic perturbations. *Chaos Solitons Fractals*, **152**: 111338, 2021.

Climate system : A global sensitivity approach

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Abstract: This article is a first attempt to develop a numerical approach to solving differential equations based on Galerkin projections and extensions of polynomial chaos to analyze the sensitivity of input parameters in the Lorenz-Stenflo climate model [1]. The sensitivity analysis [2, 3] was undertaken to measure the influence of key parameters (chemical properties of the atmosphere, rotation,temperature gradient, convection motion). In addition, we do simulations of the climate model in the non-chaotic case and in the chaotic case and we calculate the Sobol's indices when the parameters follow the uniform law.

Keywords: Climate ; Chaotic system ; Sensitivity analysis ; Polynomials chaos ; Sobol Indice ; Lorenz-Stenflo ; Simulations.

References

[1] G. A. Leonov. Generalized Lorenz equations for acoustic-gravity waves in the atmosphere. Attractors dimension, convergence and homoclinic trajectories. *Commun. Pure Appl. Anal.*, **Vol** 16(6): 2253–2267, 2017.

[2] Ismail, Liban , Djellout, Hacène & Chauvière, Cédric. Global sensitivity analysis in the SIHR epidemiological model with application to COVID-19. *Journal of Statistics and Management Systems*, **Vol** 27:7: 2253–2267, 2017.

[3] I. M. Sobol. Global sensitivity indices for nonlinear mathematical models and their monte carlo estimates. *Math. Comput. Simulation*, **Vol** 55(1-3): 271–280, 2001.

Epidemic Threshold and Mathematical Analysis of a Stochastic SIRS Model with Cure and General Incidence Rate in a Population with Varying Size

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Abstract: Mathematical models are becoming increasingly fundamental across various fields, particularly in epidemiology, which studies the factors influencing health and disease risks within human populations. Depending on the type of differential equations used for modeling, different frameworks can analyze the spread and control of infectious diseases. The foundations of compartmental model-based epidemiological approaches were established by Sir Ronald Ross, W.H. Hamer, A.G. McKendrick, and W.O. Kermack. Our study presents a stochastic SIRS epidemic model with cure and a general incidence rate in a population with varying size. The proposed nonlinear incidence function ($I \to g(I)$) generalizes various types of transmission rates. We verify the existence of a unique global positive solution and determine a threshold $\mathcal{R}_s$ that defines the conditions for disease extinction or persistence. Furthermore, under certain assumptions, we demonstrate the existence of an ergodic stationary distribution for the system. Finally, numerical simulations are performed and discussed to validate the theoretical findings.

Keywords: Epidemic model, General rate incidence, Extinction, Persistence, Ergodicity.
2020 Mathematics Subject Classification: Probability theory and stochastic processes, 60H15, 60H30.

References

[1] Berrhazi, B. E., El Fatini, M., Laaribi, A., Pettersson, R., & Taki, R. A stochastic SIRS epidemic model incorporating media coverage and driven by Lévy noise. *Chaos, Solitons & Fractals*, **105** : 60-68, 2017.

[2] Ji, C., & Jiang, D. Threshold behaviour of a stochastic SIR model. *Applied Mathematical Modelling*, **38** , (21-22), 5067-5079, 2014.

[3] Khasminskii, R. *Stochastic stability of differential equations* (Vol. 66). Springer Science & Business Media, 2011.

[4] Liu, Q., & Chen, Q. Analysis of the deterministic and stochastic SIRS epidemic models with nonlinear incidence. *Physica A: Statistical Mechanics and its Applications*, **428**, 140-153, 2015.

[5] Mao, X. *Stochastic differential equations and applications* . Elsevier, 2007.

[6] R.Z. Hasminskii, Stochastic Stability of Differential Equations, Sijthoff and Noordhoff, Alphen aan den Rijn, The Netherlands, 1980.

[7] Zhao, Y., Jiang, D., Mao, X., & Gray, A. The threshold of a stochastic SIRS epidemic model in a population with varying size. *Discrete and Continuous Dynamical Systems-Series B*, **20** (4), 1277-1295, 2015.

A Descriptive Overview of Rammed Earth

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Abstract: Earthen materials are the first construction technology used by humanity, and the historical monuments from east to west witnessed the performance and significance of this kind of construction. The use of rammed earth has become a trend since the beginning of climate change awareness, owing to its low carbon emissions compared to other conventional materials. In particular, it has excellent thermal performance and uses local soil materials to minimize greenhouse emissions and embodied energy, making it the best choice for green buildings. This material has been the subject of several studies that have examined the mechanical behavior characteristics of rammed earth. Shear and tensile strength are weak properties of rammed earth[1], which make it prone to earthquakes. The standards and codes provide an empirical ratio between the tensile strength and compressive strength of approximately 10%[2],[3], and a ratio between the shear strength and compressive strength of roughly 7%[2],[3]. Generally, the mechanical behavior depends on numerous parameters, such as water content, soil dosage, layer thickness, wall thickness, percentage and type of stabilizer, and compaction energy. The latter is mechanical stabilization, which affects the compressive strength by enhancing the dry density[4]. Cement and lime are chemical stabilizers that are most commonly used at different percentages. To boost seismic performance, reinforced steel and wood showed high performance compared with unenforced, which could be damaged rapidly due to seismic loading. Finally, although rammed earth is a beneficial type of green building, there is little information on its mechanical behavior and many issues that need to be solved.

Keywords: Earthen materials, rammed earth, low carbon emissions, thermal performance, compressive strength, unstabilized rammed earth, stabilized rammed earth, tensile strength, shear strength, seismic performance.
2020 Mathematics Subject Classification: Primary 74C05, 74M05, 74H45.

References

[1] F. Ávila, E. Puertas, and R. Gallego. Characterization of the mechanical and physical properties of unstabilized rammed earth: A review. *Construction and Building Materials*, **270**: 121435, 2021.

[2] Royaume du Maroc. *Règlement Parasismique des Constructions en Terre (RPCTerre 2011 et RPACTerre 2011)*. Décret n° 2-12-666 du 28 mai 2013, Bulletin Officiel n° 6206, Royaume du Maroc, 2013.

[3] Standards New Zealand. *NZS-42972024 Engineering design of earth buildings*. Standards New Zealand, Wellington, New Zealand, 2024.

[4] B. V. Venkatarama Reddy. *Compressive Strength of Rammed Earth*. Editor: Springer, Singapore, 2022.

Some results on the proper strong Property ($\mathcal{A}$)

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Abstract: The main purpose of this paper is to introduce and investigate a new class of rings lying properly between the class of $\mathcal{A}$-rings and the class of $\mathcal{SA}$-rings. The new class of rings, termed the class of $\mathcal{PSA}$-rings, turns out to share common characteristics with both $\mathcal{A}$-rings and $\mathcal{SA}$-rings. Numerous properties and characterizations of this class are given as well as the module-theoretic version of $\mathcal{PSA}$-rings is introduced and studied.

Keywords: $\mathcal{A}$-module; $\mathcal{A}$-ring; idealization; $\mathcal{PSA}$-ring; $\mathcal{SA}$-module; $\mathcal{SA}$-ring; zero divisor.

References

[1] A. Ait Ouahi, S. Bouchiba and M. El-Arabi, *On proper strong Property (A) for rings and modules*, J. Algebra Appl., 19 No. 12 (2020) 2050239.

[2] D. D. Anderson and S. Chun, *The set of torsion elements of a module*, Comm. Algebra 42 (2014) 1835-1843.

[3] D. D. Anderson and S. Chun, Zero-divisors, *torsion elements, and unions of annihilators*, Comm. Algebra 43 (2015) 76-83.

[4] D. D. Anderson and S. Chun, *Annihilator conditions on modules over commutative rings*, J. Alg. Applications, Vol. 16, No. 7 (2017) 1750143 (19 pages).

[5] D. D. Anderson and S. Chun, *McCoy modules and related modules over commutative rings*, Comm. Algebra 2017, Vol. 45, NO. 6, 2593?2601.

[6] D. D. Anderson and M. Winders, *Idealization of a module*, J. Commut. Algebra 1 (2009) 3-56.

[7] A. Y. Darani, *Notes on annihilator conditions in modules over commutative rings*, An. Stinnt. Univ. Ovidius Constanta 18 (2010) 59-72.

[8] C. Faith, *Annihilator ideals, associated primes, and Kasch-McCoy commutative rings*, Comm. Algebra 19 (1991) 1867-1892.

[9] D. E. Dobbs and J. Shapiro, *On the strong (A)-ring of Mahdou and Hassani*, Mediterr. J. Math. 10 (2013) 1995-1997.

[10] C. Y. Hong, N. K. Kim, Y. Lee and S. T. Ryu, *Rings with Property (A) and their extensions*, J. Algebra 315 (2007) 612-628.

[11] J. A. Huckaba, *Commutative Rings with Zero Divisors*, Marcel Dekker, Inc., New York and Basel, 1988.

[12] J. A. Huckaba and J. M. Keller, *Annihilation of ideals in commutative rings*, Pacific J. Math. 83 (1979) 375-379.

[13] I. Kaplansky, *Commutative Rings*, Polygonal Publishing House, Washington, New Jersey,

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Notes on o-minimal structures

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Abstract: In this talk, we are going to define o-minimal structures over the real field and state some of its crucial properties and applications.

Keywords: O-minimal, polynomially bounded structure.
2020 Mathematics Subject Classification: Primary 03C64.

References

[1] M. Coste. *An introduction to o-minimal geometry.* Institut de Recherche Math´ematique de Rennes. November 1999.

[2] C. Miller. Infinite Differentiability in Polynomially Bounded O-Minimal Structures. Proc.Amer.Math.Soc.123(1995),2551-2555

[3] Y. Zhang. O-minimal Chow's Theorem. https://people.math.osu.edu/zhang.6100/O- MinimalChow.pdf.

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Analysis of a ψ-Hilfer Fractional Kirchhoff Equation in a New Fractional Orlicz Space

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Abstract: This paper presents a novel functional framework by defining and analyzing the ψ-Hilfer fractional Orlicz space $\mathcal{O}_G^{\alpha,\gamma,\psi}(\Lambda,\mathbb{R})$. This space extends traditional function spaces by integrating fractional calculus with the flexibility of Orlicz spaces, allowing for a broader class of function growth conditions.

A key contribution of this work is the qualitative analysis of this newly introduced space. We investigate fundamental structural properties such as reflexivity, completeness, and separability, which play a crucial role in functional analysis and the study of variational problems.

Additionally, we establish a continuous embedding of $\mathcal{O}_G^{\alpha,\gamma,\psi}(\Lambda,\mathbb{R})$ into a suitable Orlicz spaces. This result provides deeper insight into the relationship between our proposed space and existing functional frameworks, ensuring its applicability in mathematical analysis.

As a practical application, we employ Ricceri's three critical points theorem to demonstrate the existence of three weak solutions for a class fractional Kirchhoff type equations. This application underscores the effectiveness of our newly developed space in solving variational problems, particularly in the context of critical point theory.

Overall, this work bridges fractional calculus, Orlicz spaces, and variational analysis, providing a novel mathematical setting for studying complex differential equations.

Keywords: New (G,ψ)-Hilfer fractional Orlicz space, Nonlocal Kirchhoff type problem, three critical points theorem.
2020 Mathematics Subject Classification: Primary 35R11, 47G20, 35S15, 35A15.

References

[1] R. A. Adams, Sobolev spaces, Ac. Press, New York, vol. 1 (1975), pp. 975.

[2] . Almeida, A Caputo fractional derivative of a function with respect to another function, Communications in Nonlinear Science and Numerical Simulation, vol. 44 (2017), pp. 460-481.

[3] E. Azroul, A. Benkirane, and M. Srati, Existence of solutions for a nonlocal type problem in fractional Orlicz Sobolev spaces, Advances in Operator Theory, vol. 5 (2020), no 4, pp. 1350-1375.

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Dynamical behaviors of a stochastic epidemic model with the Ornstein-Uhlenbeck process

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Abstract: This study delves into a stochastic infectious disease model incorporating the Ornstein-Uhlenbeck process. Initially, we establish the theoretical existence of a unique global positive solution within the framework of this model. Subsequently, we derive suitable conditions that determine whether the disease will die out or persist.
Finally, numerical simulations illustrate the theoretical findings.

Keywords: Stochastic infectious disease model, Ornstein-Uhlenbeck process, Lyapunov function.
2020 Mathematics Subject Classification: Primary 60H10, 92D30, 37H10.

References

[1] X. Mao. *Stochastic differential equations and applications*. Elsevier: Netherlands, 2007.

[2] D. Higham & P. Kloeden. *An introduction to the numerical simulation of stochastic differential equations*. Society for Industrial and Applied Mathematics (SIAM): United States, 2017.

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Stochastic Epidemic Models with Saturated Incidence and Logistic Population Growth

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Abstract: This article explores the application of ergodic theory to understand stochastic epidemic models, focusing on their long-term behaviour under random influence. The theory allows us to analyse whether these models reach a stationary distribution, which is essential for identifying sustainable behaviour in epidemic dynamics. A stochastic epidemic framework is presented, incorporating logistic birth rates and a saturated incidence rate, in order to better understand the dynamics of certain infectious diseases. These diseases pose a threat to human health and sustainable development, impacting various aspects of daily life and holding back socio-economic progress on a global scale.

The mathematical study begins by demonstrating the existence and uniqueness of a global positive solution for the proposed model. It then discusses the conditions under which infection can persist or become extinct. In addition, the conditions necessary for the existence and uniqueness of ergodic stationary distributions among the solutions of the model are established through the development of an appropriate stochastic Lyapunov function. The theoretical results are validated by numerical simulations.

Keywords: Stochastic epidemic model, Stationary distribution, Ergodicity, Extinction, Persistence,Saturation incidence, Logistic growth.
2020 Mathematics Subject Classification: Primary 60H10, 92D30, 60H30.

References

[1] Khasminskii, R.. *Stochastic stability of differential equations (Vol. 66)*. Springer Science & Business Media, 2011.

[2] Rajasekar, S.P., and M. Pitchaimani. Ergodic stationary distribution and extinction of a stochastic SIRS epidemic model with logistic growth and nonlinear incidence. *Applied Mathematics and Computation*, **377** 125143, 2020.

[3] Liu, J. Hopf bifurcation analysis for an SIRS epidemic model with logistic growth and delays. *Journal of applied mathematics and computing*, **50** (1):557–576, 2016.

Global stability of a generalized SIR Epidemic Model with Vaccination, Treatment and Distributed Delay

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Abstract: This work investigates the nonlinear dynamics of a delay-distributed SIR epidemic model incorporating a generalized incidence rate, vaccination, and treatment. The existence of two equilibrium points, along with the positivity and boundedness of the solutions, is established. To analyze the global asymptotic stability of both the disease-free and endemic equilibria, Lyapunov functionals and LaSalle's invariance principle for delay differential equations are employed. This analysis extends and refines existing results in the literature.

Keywords: SIR epidemic model; Distributed Delay; global asymptotic stability; Lyapunov functional; Vaccination, Treatment.
2020 Mathematics Subject Classification: Primary 34D20, 34D23.

References

[1] A. Abta, M. El Fatini, S. Elkhaiar, and A. Kaddar. *Global analysis for a delay-distributed SIR epidemic model* . ESAIM: PROCEEDINGS, Vol. 39, p. 1-6, 2013.

[2] A. Korobeinikov. Lyapunov Functions and Global Stability for SIR and SIRS Epidemiological Models with Non-linear Transmission. *Bulletin of Mathematical biology*, **Vol. 30** (number of the issue): 615–626, 2006.

[3] Y. Kuang . Delay Differentiel Equations With Applications in Population Dynamics, *Academic Press, INC.*, New York 1993.

Exploring the Allee Effect in a Within-Host Bacterial Infection Model

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Abstract: In this study, we propose and analyze a within-host bacterial infection model that incorporates the Allee effect, considering two scenarios: with and without an immune response. The Allee effect, a biological phenomenon where population growth is hindered at low densities, plays a significant role in understanding pathogen dynamics and treatment outcomes. Through rigorous mathematical analysis, we demonstrate the existence of up to three endemic equilibria under different parameter regimes, reflecting the complex interplay between infection pressure and host responses. We derive the basic reproduction number R_0 and reveal the presence of a backward bifurcation when $R_0 < 1$, indicating that disease persistence can occur even when the reproduction number is below unity. Additionally, the model exhibits the classical forward bifurcation as well as Hopf bifurcations, leading to oscillatory dynamics and the emergence of limit cycles. These findings highlight the critical role of initial conditions and immune response in shaping the infection outcome, and underscore the necessity of considering nonlinear population effects such as the Allee effect in within-host models.

Keywords: Within-host model, Backward bifurcation, Allee effect, ODEs system.
2020 Mathematics Subject Classification: Dynamical systems in biology, Stability theory for ODEs, Bifurcation theory.

References

[1] C. Castillo-Chavez and B. Song. Dynamical models of tuberculosis and their applications. *Mathematical Biosciences & Engineering*, **1** (2): 361–404, 2004.

[2] H.R. Thieme. *Mathematics in Population Biology*. Princeton University Press: USA, 2018.

[3] D. Aldila, J.P. Chávez, K.P. Wijaya, N.C. Ganegoda, G.M. Simorangkir, H. Tasman, and E. Soewono. A tuberculosis epidemic model as a proxy for the assessment of the novel M72/AS01E vaccine. *Communications in Nonlinear Science and Numerical Simulation*, **120**: 107162, 2023.

[4] J.C. Kamgang and G. Sallet. Global asymptotic stability for the disease-free equilibrium for epidemiological models. *Comptes Rendus. Mathématique*, **341** (7): 433–438, 2005.

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About the uniqueness of approximate numerical solutions of scalar conservation laws with a non Lipschitz flux function in an infinite space domain

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Abstract: In this note, we investigate about discrete entropy solution of scalar conservation law. We establish uniqueness of finite volume approximate solution to scalar conservation laws with a non Lipschitz flux function in the whole space. Our arguments are based on properties of moduli of continuity of the components of the numerical flux.

Keywords: Scalar conservation laws, Finite volume scheme, Modulus of continuity, Entropy formulation.
2020 Mathematics Subject Classification: Primary 65M08, 35L02.

References

[1] M. Karimou Gazibo. Degenerate parabolic equation with zero flux boundary condition and its approximations. *WSEAS Transactions on mathematics*, **23**: 682-705, (2024).

[2] Andreianov, B. and Karimou Gazibo, M., Entropy Formulation of Degenerate Parabolic Equation with Zero-Flux Boundary Condition. *Zeitschrift fur Angewandte Mathematik und Physik*, **64**, 1471-1491, (2013).

[3] Ph. Bénilan, S.N. Kruzkov, Conservation laws with continuous flux functions,*Nonlinear Differential Equations and Applications (NoDEA)*, **3**, 395-419, (1996).

Bifurcation Analysis of an Age-Structured HIV Model Incorporating viral loss, Latent Infection and Logistic Growth

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Abstract: To better understand the within-host dynamics of viral infections, including HIV, we develop an age-structured model that captures interactions between the virus and two distinct cell populations. This model incorporates a latent reservoir in the second compartment, a loss term in the free virus equation, and a logistic growth term for healthy cells. Using the method of characteristics, we reduce the system to a set of differential equations with distributed delay. A comprehensive mathematical analysis is conducted, including stability and bifurcation analysis, to identify key factors influencing disease progression. We determine the basic reproduction number, which dictates whether the infection persists or is eradicated. Furthermore, we establish sufficient conditions for the local and global asymptotic stability of the disease-free equilibrium and identify bistability phenomena. By examining the endemic equilibrium through a fourth-order exponential polynomial characteristic equation, we detect Hopf bifurcations on both the forward and upper branch of the backward bifurcation. Finally, numerical simulations validate the theoretical findings.

Keywords: HIV,Viral loss, Distributed Delay, Hopf Bifurcation
2020 Mathematics Subject Classification:92D30,37N25

References

[1] Hmarrass.H, Qesmi.R, Global stability and Hopf bifurcation of a delayed HIV model with macrophages, CD4+T cells with latent reservoirs and immune response. EPJ, 2025(Accepted).

[2] Hmarrass.H, Qesmi.R, Stability and backward bifurcation for an HIV model with macrophages and CD4+T cells with latent reservoirs. MATCOM, 2024; 225:370–395. https://doi.org/10.1016/j.matcom.2024.05.026.

[3] Qesmi.R, et al, A Hepatitis B and C Virus Model with Age since Infection that Exhibits Backward Bifurcation. SIAM J Appl Math. 2011;71(4):1509-1530. doi:10.1137/10079690X

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Approximate controllability for impulsive neutral semilinear evolution equations with nonlocal conditions

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Abstract: This work investigates the approximate controllability of a semilinear neutral-type system incorporating nonlocal conditions, delays in the state variable, and impulsive effects. We demonstrate that such influences, when treated as perturbations of an approximately controllable linear system, do not disrupt the system's controllability under suitable conditions. Our analysis is based on the theory of sectorial operators, the compactness of the semigroup generated by the linear part of the system, and an application of Rothe's fixed point theorem. The results establish that approximate controllability is preserved even in the presence of small nonlinearities and impulses, reinforcing the natural conjecture that real-world systems—often subject to such phenomena—maintain controllability despite their omission in idealized models. To illustrate the practical relevance of our approach, we examine a specific example encompassing a broad class of systems, including a neutral-type heat equation.

Keywords: Approximate controllability, Neutral semilinear evolution equations, Nonlocal conditions, Rothe's fixed point theorem
2020 Mathematics Subject Classification: Primary 34K35,93B05,34K40,35R12.

References

[1] FR. P. Agarwal, H. Leiva, L. Riera, S. Lalvay. Existence of solutions for impulsive neutral semilinear evolution equations with nonlocal conditions. *Discontinuity Nonlinearity Complex*, **2** 1–18, 2022.

[2] X. Fu. Approximate controllability for neutral impulsive differential inclusions with nonlocal conditions. *J. Dyn. Control Syst.*, **17** 359–386, 2011.

[3] FG. Isag. On Rothe's fixed point theorem in general topological vector spaces. *An. St. Univ. Ovidius Const*, **12** 127–134, 2004.

[4] C. Duque, H. Leiva, T. Karite. Approximate controllability for impulsive neutral semilinear evolution equations with nonlocal conditions. *IMA Journal of Mathematical Control and Information*, **2025** 1–25,

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Study of a Stochastic SEIR Epidemic Model with Lévy Jumps and a saturated incidence rate

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Abstract: In this paper, we consider a stochastic model of a susceptible exposed infectious recovered epidemic (SEIR) with a saturated incidence rate under the influence of Lévy jumps. We aim to study the dynamics of a system of three stochastic differential equations. We first establish sufficient conditions for the existence and uniqueness of positive solutions of the stochastic differential system starting from a positive initial value by constructing an appropriate Lyapunov function, and then obtain sufficient conditions for the extinction of the infectious disease on average. Finally, a discussion and numerical simulation are presented to verify the obtained results.

Keywords: SEIR model, Itô's formula; extinction; Lévy jumps, Euler scheme.
2020 Mathematics Subject Classification: 60G15, 60G51, 60H05.

References

[1] Ding Y, Fu Y, Kang Y. *Stochastic analysis of COVID-19 by a SEIR model with Lévy noise.* Chaos. 2021 Apr;31(4):043132. doi: 10.1063/5.0021108. PMID: 34251226.

[2] G. Ranjith Kumar, K. Ramesh, and Kottakkaran Sooppy Nisar. *Dynamics of an SEIR epidemic model with saturated incidence rate including stochastic influence.* Computational Methods for Differential Equations, 12, 2, (2024); 350-360.

[3] Ben Said. M, Aghoutane. N, Azrar. L, *A Lévy process approach coupled to the stochastic Leslie-Gower model*, MMC. (2024) ;11, 1 : pp. 178-188.

Stress phenomena with optimal control: Mathematical model and simulation

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Abstract: This work presents a new mathematical model that encompasses the dynamics of stress, the impact of workload, secondary symptoms and resilience [1]. The model introduces two control variables: $u(t)$, modulating the influence of workload, and $v(t)$, enhancing resilience. A system of non-linear differential equations describes the interaction between these factors. The framework is based on optimal control strategies to minimize stress levels and sustainably improve resilience [2, 3]. Numerical simulations demonstrate the effectiveness of these strategies, highlighting the potential of specific interventions to alleviate stress.

Keywords: Stress, Mathematical modeling, Optimal control theory.
2020 Mathematics Subject Classification: 92C20.

References

[1] Y. Khouldi, N. Idrissi Fatmi, A. Abdelalim, A. El Azri. Mathematical modeling and simulation of stress phenomena for medical students with optimal control. *Commun. Math. Biol. Neurosci.*, **2025** (53): 1–15, 2025.

[2] A. Abdelalim, N. Idrissi Fatmi, Y. Khouldi. Analysis and optimal control time for a continuous panic propagation in in-flight scenarios. *Commun. Math. Biol. Neurosci.*, **2024** (53): 1–15, 2024.

[3] M. Lafif, A. Ghazaoui, M. Rachik. Study of an optimal control model for the panic propagation in a flying aircraft. *Commun. Math. Biol. Neurosci.*, **2022**: 91, 2022.

Parabolicity and Regularity in Volterra Integro-Differential Equations

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Abstract: This communication focuses on a class of linear Volterra integro-differential equations that arise in the modeling of phenomena with memory effects. We investigate parabolic Volterra equations in Banach spaces and analyze their qualitative behavior through resolvent operator theory. A central aspect of this study is the characterization of parabolicity, a property that ensures the boundedness of the resolvent in the complex right half-plane. Under suitable assumptions on the memory kernel, we establish the existence of mild solutions with maximal regularity in Hölder-type function spaces. While the analysis remains theoretical, these equations are known to appear in time-dependent systems with memory, motivating a deeper understanding of their structural properties.

Keywords: Volterra equations, parabolicity, resolvent analysis, maximal regularity, integro-differential systems
2020 Mathematics Subject Classification: Primary 45D05, 47D06, 35R09.

References

[1] J. Prüss. *Evolutionary Integral Equations and Applications*. Birkhäuser, Monographs in Mathematics, vol. 87, Switzerland, 1993.

[2] K.-J. Engel and R. Nagel. *One-Parameter Semigroups for Linear Evolution Equations*. Springer, Graduate Texts in Mathematics, vol. 194, Germany, 2000.

[3] A. Fadili and H. Bounit. On the Complex Inversion Formula and Admissibility for a Class of Volterra Systems. *International Journal of Differential Equations*, vol. 2014, Article ID 948597, 13 pages, 2014.

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Analysis of Mild Solutions for Fractional Differential Inclusion with Nonlocal Conditions Involving the $\psi-$Caputo Derivative

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Abstract: This is concerned with the existence of mild solutions for ψ-fractional differential inclusions with nonlocal conditions in Banach spaces. The results are obtained by using fractional calculus, Hausdorff measure of noncompacteness, and the multivalued fixed point theorem.

Keywords: fractional differential inclusions, Hausdorff measure of noncompactness.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] Tingting Lian, Changfeng Xue and Shaozhong Deng. *Boundary Value Problems*. 2016.

[2] Wang, J, Zhou, Y.A class of fractional evolution equations and optimal controls. *Nonlinear Anal*, **12**: 262–272 , (2011).

[3]] Sabatier, J, Agrawal, OP, Tenreiro Machado, JA. Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering. *Springer, Dordrecht* 2007.

2nde International Workshop on Advance in Nonlinear Dynamical Systems, Complex Networks and Applications (DySyX'2025)
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$\psi-$Caputo Fractional Derivative in Coupled Nonlinear Impulsive Hybrid Differential Systems with Mixed Perturbations

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Abstract: In this work, we prove the existence and uniqueness of solutions to impulsive coupled system of nonlinear hybrid fractional differential equations involving $\psi-$Caputo fractional derivative of order $\alpha \in (0,1)$ with linear and nonlinear perturbations. We prove our main results by applying the nonlinear alternative of Leray-Schauder type and Banach's fixed point theorem. As application, on example is included to show the applicability of our results.

Keywords: Coupled systems, Fractional derivative and Fixed point theorems.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] R. Almeida. A Caputo fractional derivative of a function with respect to another function. *Nonlinear Sci. Numer. Simul*, **15** 460–481, 2017.

[2] M. Hannabou, M. Bouaouid and K. Hilal. Coupled System of Nonlinear Impulsive Hybrid Differential Equations With Linear And Nonlinear Perturbations. *TWMS J. App And Eng. Math*, **15**: 1434-1450, 2024.

[3] H. Gu, Y. Zhou, B. Ahmad and A. Alsaedi. Integral solutions of fractional evolution equations with nondense domain. *Electronic journal of differential equations*, **145**: 1–15, 2017.

Adaptive Clustering Through Fractional Probabilistic Self-Organizing Maps and Genetic Optimization

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Abstract: This study introduces an innovative clustering technique that integrates fractional calculus with a probabilistic self-organizing map and genetic algorithm-based optimization. By addressing the influence of memory in data and adaptively tuning the fractional order parameter α, the method improves clustering performance. Experimental results demonstrate superior outcomes compared to conventional approaches across multiple datasets, highlighting its potential for applications like image compression.

Keywords: Fractional derivative, Probabilistic self-organizing map, Genetic algorithm, Clustering
2020 Mathematics Subject Classification: Primary 34A08, 68T05, 90C59.

References

[1] T. Alam, S. Qamar, A. Dixit, and M. Benaida. Genetic algorithm: reviews, implementations, and applications. *arXiv preprint arXiv:2007.12673*, 2020.

[2] A. Atangana and D. Baleanu. New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. *arXiv preprint arXiv:1602.03408*, 2016.

[3] S. S. Cheng, H. C. Fu, and H. M. Wang. Model-based clustering by probabilistic self-organizing maps. *IEEE Transactions on Neural Networks*, **20**(5): 805–826, 2009.

A stability result for a nonlinear Neumann boundary value problem : probabilistic approach

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Abstract: We consider a sequence of partial differential equations (PDEs) with a nonlinear Neumann boundary condition. When the sequences of coefficients converge to appropriate functions, the viscosity solution of the initial problem converges to a viscosity solution of PDEs with limit coefficients. We use stability arguments of a reflected stochastic differential equation to establish the convergence of the considered viscosity solution sequence. Our approach is based on the theory of backward stochastic differential equations..

Keywords: Reflecting stochastic differential equation, Backward stochastic differential equations, Stability, Neumann condition, Partial differential equation, Viscosity solution.
2020 Mathematics Subject Classification: 60H99, 60H30, 35K61.

References

[1] K. Bahlali, L. Maticiuc, A. Zalinescu. Penalization method for a nonlinear Neumann PDE via weak solution of reflected SDEs, Electron. J. Probab. 18, no. 102, 1-19, 2013.

[2] K. Bahlali, B. Boufoussi, S. Mouchtabih. Penalization for a PDE with a nonlinear Neumann boundary condition and measurable coefficients. Stochastics and Dynamics, 2021. https://doi.org/10.1142/S0219493721500532

[3] K. Bahlali, B. Boufoussi, S. Mouchtabih. Approximation of a degenerate semilinear PDE with a nonlinear Neumann boundary condition. Electron. J. Probab. 27 1 - 21, 2022.

[4] H. Kaneko, S. Nakao. A note on approximation for stochastic differential equations. Séminaire de Probabilité (Strasbourg). XXII, 155-162, 1988.

[5] E. Pardoux, S. Zhang. Generalized BSDEs and nonlinear Neumann boundary value problems. Prob. Theory and Rel. Fields. 110, 535-558, 1998.

[6] E. Pardoux, A. Rascanu. Continuity of the Feynman-Kac formula for a generalized parabolic equation. Stochastics (89) 5, 726-752 (2017). http://dx.doi.org/10.1080/17442508.2016.1276911

[7] A. Rozkosz, L. Slominski. On stability and existence of solutions of SDEs with reflection at the boundary. Stoc. Proc. Appl. 68, 285-302, 1997.

[8] D.W. Stroock, S.R.S. Varadhan. Diffusion Processes with boundary conditions. Comm. Pure Appl. Math. 24 , 147-225, 1971.

Global Well-Posedness For The Critical 2d Dissipative Quasi-Geostrophic Equation

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Abstract: We present an elementary proof of the global well-posedness for the critical 2D dissipative quasi-geostrophic equation. Our approach relies on a nonlocal maximum principle applied to suitable moduli of continuity.

Keywords: Well posedness, 2D dissipative quasi-geostrophic equation.
2020 Mathematics Subject Classification: Primary: 35Q35; Secondary: 76U05.

References

[1] A. Cordoba and D. Cordoba, *A maximum principle applied to quasi-geostrophic equations*, Commun. Math. Phys. 249 (2004), 511–528

[2] S. Resnick,*Dynamical problems in nonlinear advective partial differential equations*, Ph.D. Thesis, University of Chicago, 1995

[3] J. Wu, *The quasi-geostrophic equation and its two regularizations*, Comm. Partial Differential Equations 27 (2002), 1161–1181.

Comparison Principales With Applications To Linear And Nonlinear Power Fractional Differential Equations

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Abstract: This study provides some essential principle comparison theorems related to the power fractional calculus intending to analyze linear and nonlinear power fractional differential equations.

Keywords: Principle comparison, non-singular kernel, Mittag-Leffler function, fractional differential equations.

References

[1] K. Hattaf, A new generalized definition of fractional derivative with non-singular kernel, Computation 8 (2020), no. 2, Art. 49. https://doi.org/10.1155/2021/1580396

[2] E. M. Lotfi , H. Zine, D. F. M. Torres, N. Yousfi, The Power Fractional Calculus: First Definitions and Properties with Applications to Power Fractional Differential Equations. Mathematics 8 (2022), no. 10(19), 3594. https://doi.org/10.3390/math10193594

[3] H. Zitane, D. F.M. Torres, A class of fractional differential equations via power non-local and non-singular kernels: Existence, uniqueness and numerical approximations, Physica D: Nonlinear Phenomena, 457 (2024), 133951. https:https://doi.org/10.1016/j.physd. 2023.133951

Viability for a class of partial differential equations with non dense-domain

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Abstract: Let E be a Banach space, $A : D(A) \subset E \longrightarrow E$ be a linear operator on E. Let D be a locally closed subset of E and $F : [0, +\infty) \times \mathcal{B} \longrightarrow E$ a continuous function. The goal is to establish conditions under which the subset D remains invariant under the evolution described by the following functional differential equation with infinite delay

$$\begin{cases} \frac{dv(t)}{dt} = Av(t) + F(t, v_t), & \text{for } t \geqslant 0, \\ v_0 = \phi \in \mathcal{B}, \end{cases}$$

In other words, if the initial state of the system starts in D, then the system's trajectory will stay within D for all future times.
A sufficient condition for the invariance of D is known as the "tangency condition." This requires that there exists a bounded linear operator B that interacts with A in a specific way. Essentially, for small changes over time, solutions initiated within D should remain close to D, with a particular rate of approach dictated by both A and B. This tangency condition ensures that solutions do not "escape" D within a certain time frame, as long as the initial conditions satisfy certain bounds.

Keywords: Invariant sets, Mild solutions, Partial functional differential equations, Tangency condition
2020 Mathematics Subject Classification: Dynamical systems

References

[1] D. Montreanu ; N. Pavel *Tangency, flow invariance for differential equations and optimization problems* . Marcel Dekker: New York , 1999.

[2] K.EZZINBI ; Y. STAILI. Invariant sets for a class of semilinear delay differential equations with non-dense domain. *J. Math. Anal. App*, **Vol 528** (2), 2023.

[3] K.EZZINBI ; Y. STAILI. Viability for a class of partial differential equations with non-dense domain. *J. Math. Anal. App*, **Vol 548** (1), 2025.

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Stability analysis of an extended age stratified SEIRS model with cases in intensive unit care

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Abstract: This work investigates the global stability of an extended SEIRS model incorporating hospitalized and intensive care unit (ICU) cases. First, we analyze the model without vital dynamics, reducing it to a Kermack-McKendrick-type framework, and examine its mathematical properties. Next, we introduce immunity waning to study the resulting dynamical behavior. To establish the global stability of equilibria—particularly the endemic equilibrium—we combine Lyapunov function theory with the stable Volterra Lyapunov matrix concept. This approach enables us to determine the eigenvalue signs of a high-dimensional matrix through a numerical conjecture. Our results demonstrate that the disease-free equilibrium is globally asymptotically stable when the basic reproduction number satisfies $\mathcal{R}_0 \leq 1$, while the endemic equilibrium is globally asymptotically stable for $\mathcal{R}_0 > 1$.

Keywords: Kermack-McKendrick, Global Stability ,Volterra-Lyapunov matrix
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] P.Magal, O.Seydi, G.Webb *Final size of a multi-group SIR epidemic model: Irreducible and non-irreducible modes of transmission*, Mathematical Biosciences, Volume 301, 2018.

[2] Shu Liao, Jin Wang, *Global stability analysis of epidemiological models based on Volterra-Lyapunov stable matrices*, Chaos, Solitons and Fractals Volume 45, Issue 7, July 2012, Pages 966-977.

[3] H. Salje, C. Tran Kiem, N. Lefrancq, N. Courtejoie, P. Bosetti, J. Paireau, A. Andronico, N. Hozé, J. Richet, CL. Dubost, Y. Le Strat, J. Lessler, D. Levy-Bruhl, A. Fontanet, L. Opatowski, PY. Boelle and S. Cauchemez. *Estimating the burden of SARS-CoV-2 in France.* ,Science, 2020.

Existence and multiplicity of positive solutions for an elliptic system deriving from strongly coupled critical terms and concave nonlinearities

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Abstract: In this paper, we study the existence of non-trivial multiple solutions for an elliptic system with strongly coupled critical terms and concave nonlinearities in bounded domains. Using the variational method, especially the Nehari manifold and Palais–Smale condition, we prove the existence and multiplicity results of positive solutions.

Keywords: Elliptic system, strongly coupled critical terms, positive solutions, Nehari manifold, variational method
2020 Mathematics Subject Classification: Primary 35B33, 35J50, 35J57.

References

[1] R. Echarghaoui, A. Hatimi, M. Hatimi, A Fractional Elliptic System With Strongly Coupled Critical Terms and Concave-Convex Nonlinearities, Int. J. Anal. Appl., **22** (2024), 107. https://doi.org/10.28924/2291-8639-22-2024-0

[2] M. Hatimi, R. Echarghaoui, A p-Laplacian Elliptic System with Strongly Coupled Critical Terms and Concave-Convex Nonlinearities, Int. J. Anal. Appl., **23** (2025), 23. https://doi.org/10.28924/2291-8639-23-2025-23

[3] X. Zhou, HY. Li and JF. Liao, Multiplicity of Positive Solutions for a Semilinear Elliptic System with Strongly Coupled Critical Terms and Concave Nonlinearities. Qual. Theory Dyn. Syst. **22**, 126 (2023). https://doi.org/10.1007/s12346-023-00825-9

Stochastic Dynamics of a Heroin Use Model with Relapse and General Nonlinear Incidence Rate

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Abstract: In this work, we adopt the assumption that heroin use can be modeled similarly to the spread of an infectious disease. We develop a heroin epidemic stochastic model with a nonlinear incidence rate. We begin by proving that the system admits a unique global positive solution. Next, we derive criteria for the extinction and persistence of heroin users within this stochastic framework. Furthermore, we establish the existence of a unique ergodic stationary distribution for the system. Finally, numerical simulations are presented to illustrate the effectiveness of the theoretical results.

Keywords: Stochastic epidemic model; Extinction and permanence in mean; stationary distribution;
2020 Mathematics Subject Classification: 93B70, 91G30.

References

[1] Ruan, Shigui and Wang, Wendi *Dynamical behavior of an epidemic model with a nonlinear incidence rate*, Journal of differential equations, 135–163,2003, Elsevier.

[2] Wang, Lianwen and Zhang, Xingan and Liu, Zhijun *An SEIR epidemic model with relapse and general nonlinear incidence rate with application to media impact*; Qualitative Theory of Dynamical Systems, 309–329,2018, Springer.

[3] Liu, Shitao and Zhang, Liang and Zhang, Xiao-Bing and Li, Aibing *Dynamics of a stochastic heroin epidemic model with bilinear incidence and varying population size*;International Journal of Biomathematics;2019.

[4] Chen, Tao and Li, Zhiming *Dynamics and stationary distribution of a stochastic SIRS epidemic model with a general incidence and immunity*;Boundary Value Problems,85,2022,

[5] White, Emma and Comiskey, Catherine, *Heroin epidemics, treatment and ODE modelling*, Mathematical biosciences, 2007, 208(1), 312–324, Elsevier.

Moment method estimation for stochastic differential equations with random effects based on continuous observations

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Abstract: We consider a stochastic differential equations driven by a fractional Brownian motion with Hurst index $H \in (0,1)$, with drift term relying linearly on some random effects ϕ_i. The distribution of the random effect ϕ depends on unknown population parameter θ wich is to be estimated. In this paper, We propose a moment method of estimation of this parameter and we examine the consistency and normality asymptotic properties of the obtained estimators.The performance of the estimation method is illustrated by numerical simulations. An empirical application is made to the real Asian financial data..

Keywords: Geometric Fractional Model, Random effects, Method of moments, Stock prices.
2025 Mathematics Subject Classification: Primary 91B70 , 60G22, 62G07, 62G20, 91G15.

References

[1] Y.Mishura. *Stochastic calculus for fractional Brownian motion and related processes*, volume 1929. Springer Science and Business Media, 2008.

[2] B.Mandelbrot and J.Ness. *Fractional brownian motions, fractional noises and applications.* SIAM review, 10(4):422-437, 1968..

[3] N.Etemadi. *An elementary proof of the strong law of large numbers.* Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete, 55(1):119-122, 1981.

[4] M.Delattre, V.Catalot, and A.Samson. *Estimation of population parameters in stochastic differential equations with random effects in the diffusion coefficient.* ESAIM: Probability and Statistics, 19:671-688, 2015.

[5] M. El Omari, H. El Maroufy, and C.Fuchs. *Non parametric estimation for fractional diffusion processes with random effects.* Statistics, 53(4):753-769, 2019

[6] A.Rascanu et al. Differential equations driven by fractional brownian motion. Collectanea Mathematica, pages 55-81, 2002.

Global Behavior, Extinction, and Persistence of Solutions to a Fast Diffusion p-Laplace Equation with Logarithmic Nonlinearity

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Abstract: In this paper, we study a class of fast diffusion Kirchhoff-type p-Laplace equation with logarithmic nonlinearity. Under appropriate conditions, by applying energy estimates in combination with the Galerkin method and Sobolev inequality, we establish the global existence of solutions. Moreover, we obtain the extinction and non-extinction properties of these solutions.

Keywords: p-Laplace equation, Kirchhoff-type models, logarithmic nonlinearity, global existence, extinction.
2020 Mathematics Subject Classification: Primary 35K65, 35B44, 35B40..

References

[1] T. Boudjeriou, Global Existence and Blow-Up for the Fractional p-Laplacian with Logarithmic Nonlinearity, *Mediterr. J. Math.*, **17**(5), 2020, 162.

[2] H. Brézis, *Functional Analysis. Theory and Applications*, Collection Mathématiques Appliquées pour la Maîtrise, 1994.

[3] H. Chen, P. Luo, G. Liu, Global Solution and Blow-Up of a Semilinear Heat Equation with Logarithmic Nonlinearity, *J. Math. Anal. Appl.*, **422**(1), 2015, pp. 84–98.

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A meshfree approach using locally supported radial basis functions for 3D advection-diffusion and 2D shallow water equations with Neumann boundaries

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Abstract: This work presents a novel application of locally supported radial basis functions (LSRBFs) for the numerical solution of complex partial differential equations arising in environmental and geophysical modeling. Specifically, the method is applied to the 3D advection-diffusion equation:

$$\frac{\partial u}{\partial t}+\left(\beta_x\frac{\partial u}{\partial x}+\beta_y\frac{\partial u}{\partial y}+\beta_z\frac{\partial u}{\partial z}\right)-\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=r$$

where β_x, β_y, β_z are the velocity components of the carrier fluid along the axes (x, y, z) respectively and r the source term; and 2D shallow water equation

$$\frac{\partial \boldsymbol{K}}{\partial t}+\frac{\partial \boldsymbol{J}}{\partial x}+\frac{\partial \boldsymbol{P}}{\partial y}=\boldsymbol{M}$$

where the vectors K, J, P and M are given by:

$$\boldsymbol{K}=\begin{pmatrix}\eta\\ u\\ v\end{pmatrix};\qquad \boldsymbol{J}=\begin{pmatrix}Hu\\ g\eta\\ 0\end{pmatrix};\qquad \boldsymbol{P}=\begin{pmatrix}Hv\\ 0\\ g\eta\end{pmatrix};\qquad \boldsymbol{M}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

the total water depth h represents the sum of the total water depth η and the mean water depth H. The unknown amplitudes to be determined are η, u, and v.
Both of which model key physical processes such as pollutant transport and water flow dynamics. The proposed meshfree approach leverages the localized nature of the radial basis functions to enhance computational efficiency and stability. This approach converts the concerned PDEs models into a system of ordinary differential equations, which are then solved numerically Special attention is given to the treatment of Neumann boundary conditions, which are naturally incorporated into the formulation without requiring additional modifications. Numerical experiments demonstrate the accuracy and robustness of the method, highlighting its potential for large-scale, real-world simulations.

References

[1] Lanceni Keita, Lahcen Azrar, Ateq Ahmed Al-Ghamedi. A local meshfree approach based on compactly supported radial basis functions for 2D coupled fluid dynamics PDEs on regular and irregular domains.*Engineering Analysis with Boundary Elements*, **177** (106246), 2025.

Global dynamics of a within-host viral model with immune response and two delays

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Abstract: In this work, we study the asymptotic behaviour of the homogeneous in space solution of a delay differential equation arising from a viral infection model with immune response involving two different delays . Existence and smoothness of solutions are investigated. Then, using the monotone dynamical systems stability framework, sufficient conditions providing asymptotic and global stability of the free-infection and the endemic equilibriums are derived, depending on the delay and the function representing the efficiency of immune response-mediated virus elimination. Afterwords, clinical data are used to calibrate the differential equation and illustrate the analytical results by numerical simulations with the obtained parameters values.

Keywords: Immune response mathematical modeling, Delay Differential Equations, Monotone dynamical systems, Global stability, Asymptotic stability
2020 Mathematics Subject Classification: 34K20, 37N25.

References

[1] G. Bocharov, A. Meyerhans, N. Bossonov, S. Trofimchuk and V.Volpert, Modelling the dynamics of virus infection and immune response in space and time, *International Journal of Parallel, Emergent and Distributed Systems*, (2017), 1-15.

[2] G. Bocharov, Modelling the dynamics of LCMV infection in mice: conventional and exhaustive CTL responses. *J. Theor. Biol*, **192**, (1998), 283-308.

[3] J. El Karkri and K. Niri, Global asymptotic stability of an SIS epidemic model with variable population size and a delay, *Int. J. Dynamical Systems and Differential Equations*, **7**, No 4 (2017), 289-300.

[4] H.L. Smith, Monotone dynamical systems: An introduction to the theory of competitive and cooperative systems, *Mathematical Surveys and Monographs*, **41**, Amer. Math. Soc. Providence, (1995).

A Strongly Coupled Sub-Laplacian System on the Heisenberg Group $\mathbb{H}_1$

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Abstract: In this work, we study the following sub-elliptic system involving strongly coupled critical terms and concave nonlinearities:

$$\begin{cases} -\Delta_{\mathbb{H}_1} u = \dfrac{\eta_1\alpha_1}{2^*}|u|^{\alpha_1-2}|v|^{\beta_1}u + \dfrac{\eta_2\alpha_2}{2^*}|u|^{\alpha_2-2}|v|^{\beta_2}u + \lambda g(z)|u|^{q-2}u, & z \in \Omega, \\ -\Delta_{\mathbb{H}_1} v = \dfrac{\eta_1\beta_1}{2^*}|u|^{\alpha_1}|v|^{\beta_1-2}v + \dfrac{\eta_2\beta_2}{2^*}|u|^{\alpha_2}|v|^{\beta_2-2}v + \mu h(z)|v|^{q-2}v, & z \in \Omega, \\ u = v = 0, & z \in \partial\Omega, \end{cases}$$

where Ω is an open bounded subset of $\mathbb{H}_1$ with smooth boundary, $-\Delta\mathbb{H}_1$ is the sub-Laplacian on Heisenberg group $\mathbb{H}_1$, $\eta_1, \eta_2, \lambda, \mu$ are positive, $\alpha_1 + \beta_1 = 2^*$, $\alpha_2 + \beta_2 = 2^*$, $1 < q < 2$, $2^* = 4$ is the critical Sobolev exponent on the Heisenberg group with $Q = 4$ the homogeneous dimension of $\mathbb{H}_1$. By exploiting the Nehari manifold and variational methods, we prove that the system has at least two positive solutions.

Keywords: Sub-Laplacian, Heisenberg group, Nehari manifold.
2020 Mathematics Subject Classification: 35H03, 35J50.

References

[1] CO. Alves , YH. Ding . Multiplicity of positive solutions for the p-Laplacian equation involving critical nonlinearity, *J. Math. Anal. Appl.* 508 521, 2003.

[2] S. Benmouloud, R. Echarghaoui, SM. Sbaï. Multiplicity of positive solutions for a critical quasilinear elliptic system with concave and convex nonlinearities. *Journal of Mathematical Analysis and Applications*. 396(1): 375-385, 2012.

[3] Birindelli I, Ferrari F, Valdinoci E. Semilinear PDEs in the Heisenberg group: the role of the right invariant vector fields, *Nonlinear Anal.*72: 987–997, 2010.

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Navigating Zero Returns: A Comprehensive Comparative Analysis of Log-GARCH and Stochastic Volatility Models in Financial Econometrics

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Abstract: This paper presents a comparative analysis of log-GARCH and stochastic volatility models, addressing volatility modeling challenges in financial econometrics with zero returns. Using QML estimation with Kalman filtering for asymmetric log-GARCH and SV models, we extend imputation methodologies from Franq [1], Sucarrat [3], and Sucarrat [4].
The log-GARCH model follows:

$$\begin{aligned} \varepsilon_t &= \sigma_t \eta_t, \eta_t \sim iid(0,1) \\ \log(\sigma_{t+1}^2) &= \alpha_0 + \alpha \log(\varepsilon_t^2) + \beta \log(\sigma_t^2) \end{aligned} \tag{1}$$

where σ_t^2 is conditional variance, ε_t is the innovation, and parameters govern volatility dynamics. This transformation ensures non-negativity and computational efficiency.
In contrast, the SV paradigm introduced by Melino and Turnbull [2] and Taylor [5] models volatility as a latent stochastic process, offering enhanced flexibility. Our framework addresses zero returns which distort parameter estimates and undermine forecasting reliability.
By comparing ALGARCH and ASV models, we provide insights into their efficacy across market conditions, extending the literature with solutions applicable to both observation-driven and parameter-driven volatility models, with implications for risk assessment and portfolio optimization.

Keywords: Log-GARCH models, Stochastic volatility, Zero returns, Volatility modeling.
2020 Mathematics Subject Classification: Primary 62P05, 91G70, 62M10.

References

[1] Francq C., Wintenberger O., Zakoian J.M. GARCH models without positivity constraints: Exponential or log GARCH?. J. Econom. 177(1): 34–46, 2013.

[2] Melino A., Turnbull S.M. Pricing foreign currency options with stochastic volatility. J. Econom. 45(1–2): 239–265, 1990.

[3] Sucarrat G., Escribano A. Unbiased QML estimation of log-GARCH models in the presence of zero returns, 2013.

[4] Sucarrat G., The Log-GARCH Model via ARMA Representations. Econom. Stat. 8, 153–169, 2018.

[5] Taylor S.J. Modeling stochastic volatility: A review and comparative study. Math. Financ. 4(2): 183–204, 1994.

Ground state solution for a critical Schrodinger equation involving the fractional p-Laplacian operator

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Abstract: In this communication, our aim is to present a generalized version of Berestycki-Lions theorem about the fractional p-Laplacian operator. As an application of this theorem, we consider the existence and regularity of solution to the critical Schrödinger equation with potential term.

Keywords: Quasilinear elliptic equations. Berestycki-Lions theorem. Schrodinger equation. Fractional p-Laplacian.
2020 Mathematics Subject Classification: Primary 35J60, 35B33.

References

[1] Alves, C., Souto, M., Montenegro, M.: Existence of a ground state solution for a nonlinear scalar field equation with critical growth, Calc. Var. Partial Differ. Equ. 43 (3-4), 537-554 (2012).

[2] Badiale, M., Guida, M., Rolando, S.: Compactness and existence results for the p-Laplace equation. Journal of Mathematical Analysis and Applications, 451(1), 345-370 (2017).

[3] Benci V. and Fortunato D.: Variational Methods in Nonlinear Field Equations: Solitary Waves, Hylomorphic Solitons and Vortices, Springer Monogr. Math., Springer, Cham, (2014).

[4] Berestycki, H., Lions, P.: Nonlinear scalar field equations, I. Existence of a ground state, Arch. Ration. Mech. Anal. 82 (4), 313-345 (1983).

An Elliptic Equation Involving Critical Hardy-Sobolev Exponents and Concave Nonlinearities

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Abstract: In this work,we prouve by an approximating argument that the following Hardy–Sobolev–Maz'ya equation with critical growth

$$\begin{cases} -\Delta u - \lambda \frac{u}{|y|^2} = \frac{|u|^{2^*(s)-2}u}{|y|^s} + \mu \mid u|^{q-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

has two disjoint and infinite sets of solutions.
provided that $N > \frac{s+2+2q}{q-1}$, where $0 \leq \lambda < \frac{(k-2)^2}{4}$ when $k > 2, \lambda = 0$, $k = 2, 2^*(s) = \frac{2(N-s)}{N-2}, 0 \leq s < 2, x = (y,z) \in \mathbb{R}^k \times \mathbb{R}^{N-k}, 2 \leq k < N, \mu > 0$ and Ω is an open bounded domain in $\mathbb{R}^N$, which contains some points $x^0 = \left(0, z^0\right)$.

Keywords: Hardy-Sobolev-Maz'ya equation; infinitely many solutions; variational methods.
2020 Mathematics Subject Classification: Primary 35J60, 35B33.

References

[1] On an elliptic equation with concave and convex nonlinearities, Proc. Am. Math. Soc. 123 (1995) 3555-3555.

[2] G. Devillanova, S. Solimini, Concentration estimates and multiple solutions to elliptic problems at critical growth, Adv. Differ. Equations. 7 (2002) 1257-1280.

p-Laplacian Problem with Multiple Hardy-Sobolev Critical Exponents in $\mathbb{R}^N$.

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Abstract: We study the partial differential equation in $\mathbb{R}^N$ involving multiple Hardy-Sobolev critical exponents. By using some interpolation inequalities, the Pohozaev identity, and an approximating problem, we prove the existence of a positive ground state solution and the regularity of the least-energy solution.

Keywords: p-Laplacian problems, Hardy-Sobolev exponents, Ground state solution.
2020 Mathematics Subject Classification: Primary 35J20, 35J60, 35J70 , 35J75, 35B09, 35B33 , 35B65.

References

[1] Bhakta, M. , Biswas, A. Hardy-Sobolev type equations for p-Laplacian, $1 < p < 2$, in bounded domain. *Differential and Integral Equations*, **24** ((1/2): 135-158, 2011.

[2] Bouabid, K., Echarghaoui, R. Infinitely Many Solutions for a Nonlinear Elliptic PDE with Multiple Hardy–Sobolev Critical Exponents. *Differ Equ Dyn Syst*, 2023.

[3] Yang, Y. The Brezis-Nirenberg problem for the fractional p-Laplacian involving critical Hardy-Sobolev exponents. arXiv preprint arXiv:1710.04654 (2017).

[4] Ghoussoub, N., Yuan, C. Multiple solutions for quasi-linear PDEs involving the critical Sobolev and Hardy exponents, *Transactions of the American Mathematical Society,Journal*, **352** : 5703-5743, 2000.

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Multiple positive solutions for mixed local and nonlocal elliptic p-Laplace equations with logarithmic and singular nonlinearities

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Abstract: In this paper, we consider a combination of local and nonlocal p-Laplace equation with logarithmic and singular nonlinearities

$$\begin{cases} -\Delta_p u + (-\Delta)_p^s u = |u|^{q-2}u \ln|u|^2 + \frac{\lambda}{u^\gamma}, & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{in } \mathbb{R}^N \backslash \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary, $0 < s < 1 < p, 0 < \gamma < 1, N > ps, 2p < q < q+2 < p_s^*, p_s^* = \frac{Np}{N-ps}$ is the critical Sobolev exponent, and $\lambda > 0$ is a real parameter. We prove the existence of two positive weak solutions to the above problem, by using the critical point theory and analytic techniques.

Keywords: mixed p-Laplace equations, fractional p-Laplacian operator, singular nonlinearity, logarithmic nonlinearity.
2020 Mathematics Subject Classification: Primary35K65, 35K55, 35B65, 35B45.

References

[1] R. Arora, VD. Rădulescu, Combined effects in mixed local–nonlocal stationary problems. Proceedings of the Royal Society of Edinburgh Section A: Mathematics. 2023:1-47. https://doi.org/10.1017/prm.2023.80

[2] K. Bal, S. Das, Multiplicity of solutions for mixed local-nonlocal elliptic equations with singular nonlinearity. arXiv preprint arXiv:2405.05832. 2024 May 9. https://doi.org/10.48550/arXiv.2405.05832

[3] S. Biagi, D. Mugnai, E. Vecchi, A Brezis–Oswald approach for mixed local and nonlocal operators. Communications in Contemporary Mathematics. 2024 Mar 4;26(02):2250057. https://doi.org/10.1142/S0219199722500572

On the convergence of Fourier-Dunkl series on the interval $[-1, 1]$

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Abstract: Some properties of the modulus of smoothness associated with the Dunkl operator are established. Using generalized Dunkl–Lipschitz conditions within Dunkl–Sobolev spaces, we derive, as a particular case, a result due to Younis on the torus. Furthermore, necessary and sufficient conditions are provided, along with a characterization of the Dini–Lipschitz classes on the interval $[-1, 1]$, in terms of the asymptotic behavior of their Fourier–Dunkl coefficients.

Keywords: Fourier–Dunkl series; Dunkl–Lipschitz conditions, modulus of smoothness
2020 Mathematics Subject Classification: Primary 42A20.

References

[1] C.F. Dunkl. Differential-difference operators associated with reflection groups. *Trans. Am. Math. Soc.*, **311** (1): 167–183, (1989).

[2] S.S. Platonov. The Fourier transform of functions satisfying the Lipschitz condition on rank 1 symmetric spaces 1. *Sib. Math. J.*, **46** (6): 1108–1118, (2005).

Analysis of Optimal Control Strategies for Multidrug-Resistant Tuberculosis

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Abstract: Multidrug-resistant tuberculosis (MDR-TB) remains a major global health challenge due to its resistance to standard treatment protocols and its high transmission potential. This work aims to investigate optimal control strategies applied to a mathematical model describing the transmission dynamics of MDR-TB. By considering the impact of various intervention strategies such as case detection and treatment adherence, we seek to identify effective control measures that can reduce the spread of the disease while minimizing associated implementation costs. To this end, an optimal control problem is formulated using suitable cost functions, and Pontryagin's Maximum Principle is employed to derive the necessary conditions for optimality. Numerical simulations are conducted to illustrate the effectiveness of the proposed control strategies. The results highlight the importance of combining multiple interventions to effectively manage the spread of MDR-TB.

Keywords: Tuberculosis, drug resistance, Optimal control, numerical simulations.
2020 Mathematics Subject Classification: 93A30, 37N25.

References

[1] Dhiraj Kumar Das, Subhas Khajanchi, T.K. Kar. The impact of the media awareness and optimal strategy on the prevalence of tuberculosis. *Applied Mathematics and Computation*, **366, 124732**. 2020.

[2] Dye, C., and Williams, B. G. Criteria for the control of drug-resistant tuberculosis. *Proceedings of the National Academy of Sciences of the United States of America*, **97(14), 8180–8185**. 2000.

[3] Kumar, A., Dubey, U. S., and Dubey, B. The impact of social media advertisements and treatments on the dynamics of infectious diseases with optimal control strategies. *Mathematics and Computers in Simulation*, **219, 50–86**. (2024).

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Conformable Spatiotemporal PDEs: Existence of Chaos and Hypercyclicity

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Abstract: This work investigates the role of conformable admissible weight functions in generating hypercyclic and chaotic behavior for the solution γ-semigroup associated with a conformable spatiotemporal partial differential equation in the weighted space $L^p_{\rho_\gamma}([0,+\infty),\mathbb{C})$. We provide sufficient conditions under which hypercyclicity and chaos occur. Additionally, leveraging a conjugacy argument, we extend our analysis to a spatiotemporal partial differential equation and establish similar results in the space $L^p_\gamma([0,+\infty),\mathbb{C})$. .

Keywords: Chaos, Hypercyclicity, Conformable PDE
2020 Mathematics Subject Classification: Primary 58F15, 58F17, 53C35.

References

[1] W. Desch, W. Schappacher, and G. F. Webb. Hypercyclic and chaotic semigroups of linear operators. *Ergodic Theory and Dynamical Systems*, **17**(4):793–819, 1997.

[2] K. G. Grosse-Erdmann and A. P. Manguillot. *Linear Chaos*. Springer Science & Business Media, London, 2011.

[3] Z. Brzeźniak and A. L. Dawidowicz. On periodic solutions to the von Foerster-Lasota equation. *Semigroup Forum*, **78**:118–137, 2009.

Effects of tumor-associated macrophages on the tumor-immune system under stochastic perturbations

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Abstract: This paper investigates the behavior dynamic of tumors with macrophage interaction. Stochastic analysis is elaborated under the environment noise in order to explore the impact of random fluctuations on the dynamics of the model. The deterministic analysis is also briefly done including stable and unstable limit cycles, period-doubling bifurcation under certain parameter conditions. Theoretical theory based on Lyapunov analysis method is elaborated to prove the existence of a unique positive solution and to determine sufficient condition for the persistence and extinction of tumor cells. Additionally, numerical simulations are elaborated to validate the theorical findings and to observe the impact of noise in the Hopf bifurcation.

Keywords: Stochastic Differential Equations, Extinction and Persistence, Tumor-macrophage

References

[1] A. BEN LAHBIB, L. AZRAR, and M. BEN AMAR. Stochastic modeling and analysis of interactions between cancer-associated fibroblasts and T-cell cytotoxic activity in tumor-immune system: the case of human lung tumors. *Submitted to Applied Mathematical Modeling*, 2025.

[2] R. EFTIMIE and C. BARELLE. Mathematical investigation of innate immune responses to lung cancer: The role of macrophages with mixed phenotypes. *Journal of Theoretical Biology*, vol. 524, p. 110-739, 2021.

[3] Y. SHU, J. HUANG, Y. DONG, and al. Mathematical modeling and bifurcation analysis of pro-and anti-tumor macrophages. *Applied Mathematical Modelling*, vol. 88, p. 758-773, 2020.

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Stability and Persistence in a two-Strain epidemic model with nonlinear transmission on heterogeneous networks

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Abstract: Motivated by the studies in [1, 2, 3], we develop a novel two-strain SIR epidemic model on heterogeneous complex networks, incorporating a nonlinear incidence rate and two distinct time delays. The model exhibits four possible equilibria: the disease-free equilibrium, the endemic equilibrium for strain 1, the endemic equilibrium for strain 2, and the coexistence equilibrium where both strains persist. Using suitable Lyapunov functions and LaSalle's invariance principle [4], we establish the global stability conditions for each equilibrium. Numerical simulations on scale-free networks are provided to illustrate and support the theoretical results.

Keywords: Epidemic model, Complex networks, Global stability, Lyapunov functions.

References

[1] Kuddus, M. A., McBryde, E. S., Adekunle, A. I., & Meehan, M. T. (2022). Analysis and simulation of a two-strain disease model with nonlinear incidence. Chaos, Solitons & Fractals, 155, 111637.

[2] CHENG, Xinxin, WANG, Yi, et HUANG, Gang. Dynamics of a competing two-strain SIS epidemic model with general infection force on complex networks. Nonlinear Analysis: Real World Applications, 2021, vol. 59, p. 103247.

[3] LI, Chin-Lung, CHENG, Chang-Yuan, et LI, Chun-Hsien. Global dynamics of two-strain epidemic model with single-strain vaccination in complex networks. Nonlinear Analysis: Real World Applications, 2023, vol. 69, p. 103738.

[4] FALL, A., IGGIDR, Abderrahman, SALLET, Gauthier, et al. Epidemiological models and Lyapunov functions. Mathematical Modelling of Natural Phenomena, 2007, vol. 2, no 1, p. 62-83.

Numerical solutions of a reaction diffusion equation using time splitting methods and application to population dynamics

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Abstract: In this paper we present a numerical approach for solving a reaction diffusion equation using a splitting method. As application we consider The Fisher–KPP equation which is used as a mathematical model for the spatio-temporal dynamics of biological populations. Our approach decomposes the original problem into two subproblems: a diffusion equation and a reaction equation, each one solved independently. The diffusion part is treated using the Fast Fourier Transform (FFT), while the reaction term is solved via a Runge–Kutta scheme combined with centered finite differences method. We analyze the convergence by studying the stability and investigating the consistency of the splitting method. Numerical simulations confirm the accuracy and robustness of this approach. The proposed method offers a simple structure, high precision, and good performance in terms of runing time.

Keywords: Fisher-KPP equation, Operator splitting method, Fast Fourier Transform, Runge-Kutta scheme.

References

[1] FISHER, Ronald Aylmer. The wave of advance of advantageous genes. Annals of eugenics, 1937, vol. 7, no 4, p. 355-369.

[2] BLANES, Sergio, DIELE, Fasma, MARANGI, Carmela, et al. Splitting and composition methods for explicit time dependence in separable dynamical systems. Journal of computational and applied mathematics, 2010, vol. 235, no 3, p. 646-659.

[3] BERESTYCKI, Henri et FANG, Jian. Forced waves of the Fisher–KPP equation in a shifting environment. Journal of differential equations, 2018, vol. 264, no 3, p. 2157-2183.

Parameter estimation of a stochastic SVIS epidemic model

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Abstract: Stochastic epidemic models play a crucial role in analyzing the spread of infectious diseases, particularly the stochastic SVIS (Susceptible–Vaccinated–Infected–Susceptible) model. These models often involve unknown parameters that can significantly affect the accuracy of predictions. Therefore, estimating these parameters is essential to improve the model's reliability.

The stochastic SVIS model divides the population into three compartments: susceptible individuals S, infected individuals I, and vaccinated individuals V. This model, as described in the work of Zhou B. et al. [1], provides a robust framework for understanding disease dynamics, forecasting outbreaks, and evaluating intervention strategies.

In this study, we estimate a few parameters of the model and the transmission states by developing an ensemble Kalman filter (EnKF) based on the stochastic SVIS epidemic model [2, 3, 4].

Keywords: Stochastic SVIS epidemic model, Ensemble Kalman filter, Estimation parameters
2020 Mathematics Subject Classification:

References

[1] B. Zhou, D. Jiang, Y. Dai, T. Hayat and A. Alsaedi. Stationary distribution and probability density function of a stochastic SVIS epidemic model with standard incidence and vaccination strategies. *Chaos, Solitons & Fractals*, **143**, 110601, 2021.

[2] G. Evensen. Sequential Data Assimilation with a Nonlinear Quasi-Geostrophic Model Using Monte Carlo Methods to Forecast Error Statistics. *Journal of Geophysical Research*, **99**, 10143–10162, 1994.

[3] R. Ghostine, M. Gharamti, S. Hassrouny and I. Hoteit. An extended SEIR model with vaccination for forecasting the COVID-19 pandemic in Saudi Arabia using an ensemble Kalman filter. *Mathematics*, **9**, 636, 2021.

[4] R. Lal, W. Huang and Z. Li. An application of the ensemble Kalman filter in epidemiological modelling. *Plos one*, **16**, e0256227, 2021.

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Implications of continuous immunity waning in a heterogeneous population

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Abstract: Our aim is to extend the SIRS mathematical epidemic model with exponential waning of immunity to allow the waning rate to differ between individuals (i.e. to introduce individual heterogeneity). Our findings are that i) compared to the classic SIRS model and SIRS models with homogeneous continuous waning, this heterogeneity increases the long-term prevalence if no prevention measure is put in place, ii) the previous models underestimate the amount of vaccine supply needed to achieve and maintain herd immunity, and iii) more vaccines are needed if the heterogeneity is unobserved (the most likely situation where vaccines are only given based on time since last vaccination) compared to when it is observed thus allowing to weak/strong immune system to affect who gets vaccinate. For parameters fitting Covid-19 (wild type strain), maintaining herd immunity requires to vaccinate individuals every 15 months according the classic SIRS model (homogeneous and sudden loss of immunity) and every 4.4 months in the most realistic model (unobserved heterogeneous and continuous loss of immunity).

Keywords: immunity, continuous waning, heterogeneity, vaccination,
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] M. El Khalifi and T. Britton. Extending susceptible-infectious-recovered-susceptible epidemics to allow for gradual waning of immunity. *Journal of the Royal Society Interface*, **20** (206): 20230042, 2023.

[2] M. El Khalifi and T. Britton. SIRS epidemics with individual heterogeneity of immunity waning. *Journal of Theoretical Biology*, **587**: 111815, 2024.

Leray-Schauder Type Solutions for a Nonlocal Problem in the New $s(\cdot,\cdot)$ - Fractional Musielak-Sobolev Space $W^{s(x,y)}L_{\Phi_{x,y}}(\Omega)$

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Abstract: The primary objective of this paper is to establish the existence of a weak solution for the following problem

$$(\mathcal{P}) \quad \begin{cases} (-\Delta)^{s(x,.)}_{a_{(x,.)}} u = \kappa(x,u) & \text{in } \Lambda, \\ u = 0 & \text{on } \partial\Lambda, \end{cases}$$

within the framework of the newly introduced $s(\cdot,\cdot)$-fractional Musielak–Sobolev spaces $(W^{s(x,y)}L_{\Phi_{x,y}}(\Omega))$. The existence result is obtained via a variational approach based on the Leray–Schauder nonlinear alternative principle.

Keywords: $s(\cdot,\cdot)$-fractional Musielak–Sobolev spaces, non-local $s(.,.)$ -order operator, Leray-Schauder's alternative theorem.
2020 Mathematics Subject Classification: Primary 35R11, 47G20, 35S15, 35A15.

References

[1] E. Azroul, A. Benkirane, M. Shimi, and M. Srati, Existence of solutions for a nonlocal type problem in fractional Orlicz Sobolev spaces. Advances in Operator Theory, vol. 5 (2020), no 4, pp. 1350-1375.

[2] .E. Azroul, A. Benkirane, M. Shimi, and M. Srati, On a class of nonlocal problems in new fractional Musielak-Sobolev spaces. Applicable Analysis, vol. 101 (2022), no 6, pp. 1933-1952.

[3] E.Azroul, A. Benkirane, M. Shimi, and M. Srati, AZROUL, Embedding and extension results in fractional Musielak–Sobolev spaces. Applicable Analysis, vol. 102 (2023), no 1, pp. 195-219.

Optimal control for the complication of type 2 diabetes and obesity:the role of awarness and treatment

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Abstract: In this study, we develop a continuous mathematical approach to model and analyze the dynamics of obesity and type 2 diabetes and their health outcome complications, in particular stroke, using the following controls: sport and diet for diabetics, education to protect diabetics from obesity, and medical treatment for stroke patients.
The purpose of our study is to minimize the number of obese people and people who have suffered a stroke, as well as the cost of controls. The Pontryagin principle is then implemented to solve optimal control problems.

Keywords: type 2 diabetes , cerebrovascular accident, obesity,optimal control,
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

References

[1] American Diabetes Association. (2003). Expert Committee on the Diagnosis and Classification of Diabetes Mellitus. *Report of the expert committee on the diagnosis and classification of diabetes mellitus. Diabetes Care, 26(1), S5-S20...* Editor: 2003, america.

[2] Kopp, R. E. (1962) *Pontryagin maximum principle. In Mathematics in Science and Engineering*, **(Vol. 5,)** (number of the issue): pp. 255-279, 1962.

[3] I. Imken, N. I. Fatmi, S. Elamari, *A new model of the spread of covid-19 among diabetes population: a mathematical analysis and optimal control approach for intervention strategies, Communications in Mathematical Biology and Neuroscience*, **Vol** (number of the issue):2023.

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Analogues of the Donoho-Stark Uncertainty Principle for Quaternion Fourier Transform

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Abstract: The classical uncertainty principle states that a nontrivial function and its Fourier transform cannot both be sharply localized. It plays an important role in signal processing and physics. in this presentation we generalizes the uncertainty principle for measurable sets from complex domain to hypercomplex domain using quaternion algebras, associated with the Quaternion Fourier transform.

Keywords: Quaternion algebras, Donoho-Stark's uncertainty principle, Signal recovery.
2020 Mathematics Subject Classification: 42A38, 44A35, 34B30

References

[1] D. L. Donoho and P. B. Stark. Uncertainty principles and signal recovery. *Journal on Applied Mathematics*, **49** (3): 906–931, 1989.

[2] K.I. Koua, Y. Yangb and C. Zou. Uncertainty Principle for Measurable Sets and Signal Recovery in Quaternion Domains. *Mathematical Methods in the Applied Sciences*, **40** (11): 3892–3900,2017.

Dynamical behavior and sensitivity analysis of a stochastic SIR epidemic model with vaccination

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Abstract: In this work, we investigate the dynamic behaviors of a stochastic model in the presence of vaccination, and perturbation caused by Gaussian white noise. First, We show the existence, uniqueness, and positivity of the solution of the model. Then sufficient conditions are presented for the extinction of the disease in terms of a threshold value R_0^S. Also, the sensitivity analysis is used to discover parameters that impact the threshold parameter. Finally, we illustrate our theoretical results by numerical simulations.

Keywords: Stochastic model; vaccination; white noise; extinction; sensitivity analysis.
2020 Mathematics Subject Classification: 34F05, 60H40, 92D30.

References

[1] X. Mao. *Stochastic differential equations and applications*. Horwood, Chichester, 1997.

[2] R. May. *Stability and complexity in model ecosystems*. Princeton University Press, Princeton, 1973.

[3] M. Semlali, K. Hattaf, M.E. El Kettani. Modeling and analysis of the dynamics of COVID-19 transmission in presence of immigration and vaccination. *Commun. Math. Biol. Neurosci.*, **Vol** (36): 1–13, 2022.

[4] Q. Liu, D. Jiang. Stationary distribution and extinction of a stochastic SIR model with nonlinear perturbation. *Appl. Math. Lett.*, **Vol** (73): 8–15, 2017.

[5] K. Hattaf, M. Mahrouf, J. Adnani, et al. Qualitative analysis of a stochastic epidemic model with specific functional response and temporary immunity. *Physica A: Stat. Mech. Appl.*, **Vol** (2018): 591–600, 2018.

[6] N.H. Du, N.N. Nhu. Permanence and extinction for the stochastic SIR epidemic model. *J. Differ. Equ.*, **Vol** (269): 9619–9652, 2020.

[7] D. Jiang, J. Yu, C. Ji, et al. Asymptotic behavior of global positive solution to a stochastic SIR model. *Math. Computer Model.*, **Vol** (54): 221–232, 2011.

[8] E. N'zi, M. N'zi. Extinction and stationary distribution of a stochastic SIRS epidemic model incorporating media coverage. *J. Adv. Math. Computer Sci.*, **Vol** (34): 1–19, 2019.

An efficient algorithm for the inverse of a block tridiagonal matrix

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Abstract: In this work we starts by discretizing the Poisson equation,in which block tridiagonal matrices are formed. Based on this, we propose a new algorithm to efficientlyinvert this class of structured matrix. We provide several numerical examples to show that the proposed algorithm is computationally efficient to illustratethe efficiency of our approach.

Keywords: Block of tridiagonal matrices, Inversion

References

[1] K. G. Bowden . *"A direct solution of the discretised form of Laplace's equation." Matrix and Tensor Quarterly.* 32, No.3, March 1982, pp. 93-96.

[2] M. Batista, A. Karawia . *The use of the Sherman–Morrison–Woodbury formula to solve cyclic block tri-diagonal and cyclic block penta-diagonal linear systems of equations.*Appl. Math. Comput. 210 (2009), pp. 558–563.

[3] B. TALIBI, A. AIAT HADj, D. SARSRI. *A numerical algorithm for computing the inverse of pentadiagonal matrix.* JAMCM 2018.

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Intelligence artificielle et neuroéducation : vers des plateformes d'apprentissage adaptatif basées sur la modélisation cognitive et la prédiction d'erreurs

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Abstract: L'intégration de l'intelligence artificielle (IA) dans le domaine de la neuroéducation représente une avancée prometteuse pour le développement de plateformes d'apprentissage adaptatif. Cet article propose une approche centrée sur la modélisation cognitive des apprenants, permettant la détection des états cognitifs (inattention, surcharge mentale, erreurs conceptuelles) à partir de données comportementales et physiologiques. L'objectif est de prédire les erreurs potentielles et d'activer des mécanismes de remédiation ciblée au sein de systèmes intelligents.

Nous présentons une architecture fonctionnelle de plateforme éducative augmentée par IA, capable d'analyser en temps réel les traces d'interaction utilisateur (réponses, temps de réaction, séquences d'activités) et d'ajuster dynamiquement les parcours pédagogiques. Une analyse multicritère fondée sur la méthode WSM (Weighted Sum Method) est appliquée pour comparer différentes stratégies de personnalisation cognitive.

Les résultats montrent que les approches hybrides combinant capteurs cognitifs et algorithmes d'apprentissage automatique permettent une adaptation plus fine et contextualisée des contenus éducatifs. Cette étude ouvre la voie à la conception de systèmes intelligents centrés apprenant, respectueux des principes de la neuroéducation et des exigences d'éthique numérique.

Keywords: Intelligence artificielle, Neuroéducation, Modélisation cognitive, Apprentissage personnalisé, Prédiction d'erreurs

Transcendence of some p adic continued fractions

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Abstract: In this contribution, we establish sufficient conditions on the elements of the p-adic continued fractions A and B which guarantee that the continued fraction A^B is a transcendental number in the p-adic framework.

Keywords: p-adic continued fraction, sufficient condition, algebraic independence.
2020 Mathematics Subject Classification:11K50, 11J81, 11J82.

References

[1] S. Ahallal and A. Kacha, Transcendental Continued Fraction, Communications in Mathematics, (30) (2022),no 1, 251-259.

[2] K. Belhroukia and A. Kacha, Transcendence and approximation measure of continued fraction, Int. J. Open Problems Compt. Math., Vol. 11, No.4, December 2018.

[3] P. Bundschuh, Transcendental Continued Fractions, J. Number Theory, 18 (1984), 91-98.

[4] A. Durand, Indépendance algébrique de nombres complexes et critére de transcendance, Composito math., no 35 (1977), 259-267.

[5] A. Kacha, Approximation algébrique de fractions continues, Thése de Doctorat d'Université de Caen, France,spécialité : Mathématiques, 1993.

[6] G. Karadeniz Gozeri, Ayetin Pekin and Adem Kiliman, On the transcendence of some power series, Advances in Difference equation, 17 (2013), 1-8.

[7] W. Lianxiang, p-adic continued fraction (II), Scientia Sinica Ser. A 28, no 10 (1985), 1018-1028.

[8] J. Liouville, Sur des classes trs ́tendues de quantités dont la valeur n'est ni algébrique, ni mme réductible des irrationnelles algébriques, C. R. Mat. Acad. Sci. Paris 18, 883-885, 910-911, (1844).